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ASSOCIATED UNIVERSITIES, INC.
UPTON, L. I., N. Y.
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DEPARTMENT OF
PHYSICS

Aug. 12, 1962

Dear Feza:

Over the last week I started to think about the problem of spin $\frac{1}{2}$ particle in a de Sitter space. The following results are of interest:

1. Let ξ^μ ($\mu = 0, 1, \ldots 4$) be real coordinates

$$\eta_{\mu\nu} \xi^\mu \xi^\nu = r^2 \qquad \text{where} \qquad \eta_{\mu\nu} = \begin{pmatrix} -1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{pmatrix}$$

and $r = \text{const.} = R$ defines the de Sitter space.

Spin $\frac{1}{2}$ eqn:

$$\left(\gamma^\mu \frac{\partial}{\partial \xi^\mu} + m\right) \psi = 0 \qquad (1)$$

and ψ = independent of r (2)

where γ^μ = constant matrices (i.e. indep. of ξ^μ)

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 \eta^{\mu\nu} \qquad (\eta^{\mu\nu} = \eta_{\mu\nu})$$

$$(\gamma^0)^\dagger = -\gamma^0, \quad \& \quad (\gamma^\mu)^\dagger = \gamma^\mu \quad \text{for } \mu \neq 0.$$

$\dagger$ = hermitian conjugate.

2. It is clear that Eqn (1) + condition (2) is invariant under the 10-parameter rotation group in the 5-dim. space

The infinitesimal rotation operators are:

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$J^\mu{}_\nu$ commutes with $\gamma^\mu \frac{\partial}{\partial \xi^\mu}$

$$J^\mu{}_\nu \equiv \xi^\mu \frac{\partial}{\partial \xi^\nu} - \eta^{\mu\alpha}\eta_{\nu\beta}\,\xi^\beta \frac{\partial}{\partial \xi^\alpha} + \frac{1}{2}\gamma^\mu \gamma_\nu$$

where $\gamma_\nu = \eta_{\nu\alpha}\gamma^\alpha$ (instead of $\xi^5 = i\xi^0$)

Remark: I use ξ^0 = real, which will be convenient later on. (This explains the $\eta^{\mu\alpha}\eta_{\nu\beta}$ factor)

3. Eqn (1) can be generated from a Lagrangian

$$\mathcal{L} \equiv \psi^\dagger \gamma^0 \gamma^\mu \frac{\partial}{\partial \xi^\mu}\psi + m\,\psi^\dagger \gamma^0 \psi \;.$$

[with, or without, condition (2)]

4. Eqn (1) plus condition (2) describes a spin $\frac{1}{2}$ particle in de Sitter space. Dirac's eqn (his 1935 Ann. of Math. article) is ugly. ~~and~~ (I ~~suggest~~ suspect) may ~~even have trouble with unitarity if it has interactions.~~

5. Eqn (1) + condition (2) gives the same result as the use of covariant derivatives:

To shorten the calculation, let me consider a 3-dim. space ($\mu = 0, 1, 2$):

squares

$$-(\xi^0)^2 + (\xi^1)^2 + (\xi^2)^2 = R^2$$

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Let $\gamma^0 = \sigma_0 = -i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\gamma^1 = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ & $\gamma^2 = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Eq'n (1) becomes:
$$\left(\sigma_0 \frac{\partial}{\partial \xi^0} + \sigma_1 \frac{\partial}{\partial \xi^1} + \sigma_2 \frac{\partial}{\partial \xi^2} + m \right) \psi = 0 \qquad (3)$$

Def'n
$$S \equiv e^{-\frac{1}{2}\left(\frac{t}{R}\right)\sigma_1} \cdot e^{-\frac{1}{2}\frac{x}{R}\sigma_0}$$

where t & x are polar coord. defined by

$$\xi^0 = r \sinh\frac{t}{R}, \qquad \xi^1 = r \cosh\frac{t}{R} \sin\frac{x}{R}$$
$$\xi^2 = r \cosh\frac{t}{R} \cos\frac{x}{R}.$$

It is easy to verify that if $\frac{\partial \psi}{\partial r} = 0$ then at $r = R$, (3) becomes

$$\left(\sigma_0 \frac{\partial}{\partial t} + \sigma_1 \frac{1}{\cosh\frac{t}{R}} \frac{\partial}{\partial x} - \frac{1}{2R} \tanh\frac{t}{R}\, \sigma_0 + m \right) S\psi = 0$$

Therefore, if $S\psi$ is a sol'n, $\sigma_2 S\psi$ is also a sol'n provided $m = 0$. [In 3-dim. σ_2 plays the role of γ^5]

The generalization to the 5-dim. space is straightforward.

We are leaving Brookhaven in two days. I will be at Stanford until Sept. 15. The address is

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Department of Physics
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After Sept. 15, I will be at Columbia. If we can find a suitable apartment in New York, most likely we will move near the end of this year.

How is the summer school of physics? Suha and you must be very busy right now. Jeannette joins me in sending you our best regards.

Yours sincerely

Tsung-Dao