To Prof. Pauli

A simple scheme for elementary particles can be constructed out of three 4-spinor fields: the neutrino ν, the electron ε and a neutral isosinglet baryon field Λ having the properties of the Λ^0 particle. It might be objected that the neutrino is in reality a two-component field. Against this it may be argued that in decays whether a neutrino is involved or not, all the particles behave like two component fields if the universal decay interaction is always of the A-V type. On the other hand a neutrino is only observed in weak processes so that we never need to take the other two components into consideration. In parity conserving strong interactions this is not so. Both right handed and left handed fields manifest themselves and if the neutrino enters the structure of particles it might be expected that the unobserved states of the neutrino nevertheless play a role in strong interactions. If neutrinos could be produced in very energetic collisions, then right handed neutrinos might be observed. The situation is similar to the electrons. If electrons ~~could~~ were only ~~th~~ emitted in weak processes we would only observe left handed electrons. But in different processes, namely the electromagnetic ones, electrons of both handedness can be produced.

The primary fields are taken to be

(1) $$\nu = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_L \\ \varepsilon_R \end{pmatrix} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \\ \Lambda_4 \end{pmatrix} = \begin{pmatrix} \Lambda_L \\ \Lambda_R \end{pmatrix}$$

Now let us represent the $\varepsilon-\nu$ system by the 2×4 matrix

(2) $$\Psi = \begin{pmatrix} \nu_L & \varepsilon_L \\ \nu_R & \varepsilon_R \end{pmatrix}.$$

In the limit of the electromagnetic forces being neglected we may take the rest mass of the electron as zero. Then Ψ satisfies the

(3) equation $$\gamma_\mu \partial_\mu \Psi = 0$$

In general the equation for Ψ is

(4) $$\gamma_\mu \partial_\mu \Psi = m\Psi \frac{1-\Sigma_3}{2}$$

where $\Sigma_3 = \gamma_1\gamma_2$, $\Sigma_1 = \gamma_2\gamma_3$, $\Sigma_2 = \gamma_3\gamma_1$ are the 4×4 spin matrices $\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$.

In the non electric limit, eq. (3) admits a 4-dimensional euclidean rotation group, or two commuting three dimensional rotation groups and two phase groups. These are

(5) $$\Psi \to \frac{1+\gamma_5}{2}\Psi e^{i\vec{\Sigma}\cdot\vec{t}} e^{i\alpha} + \frac{1-\gamma_5}{2}\Psi e^{i\vec{\Sigma}\cdot\vec{u}} e^{i\beta},$$

$u = \Sigma_3$

or, if we define the 2×2 matrices

$$\Psi_L = (\nu_L \quad \varepsilon_L) , \quad \Psi_R = (\nu_R \quad \varepsilon_R)$$

we see that Ψ_L and Ψ_R undergo independent unitary transformations to the right

(6) $$\Psi_L \to \Psi_L U = \Psi_L R e^{i(l+\nu)}, \quad \Psi_R \to \Psi_R U' = \Psi_R Q e^{i(l-\nu)}$$

The corresponding 4-spinors

(7) $$\psi_L = \begin{pmatrix} \nu_L \\ i\sigma_2 \varepsilon_L^* \end{pmatrix} \quad \text{and} \quad \psi_R = \begin{pmatrix} \nu_R \\ i\sigma_2 \varepsilon_R^* \end{pmatrix}$$

undergo independent Pauli transformations of the form

(8) $$\begin{cases} \psi_L \to e^{i\gamma_5\alpha}(a\psi_L + b\gamma_5\psi_L^c) \\ \psi_R \to e^{i\gamma_5\beta}(a'\psi_R + b\gamma_5\psi_R^c) \end{cases}$$

In a Lorentz transformation we have

$$\Psi_L \to \mathcal{L}\Psi_L , \quad \Psi_R \to \mathcal{L}^{+}\Psi_R \quad \text{or} \quad \Psi_R^{+} \to \mathcal{L}\Psi_R^{+}$$

Since we have $\Psi_R^{+} \to \Psi_R^{+}\mathcal{L}^{-1}$ we find that

$\Psi_R^{+}\Psi_L$ and $\Psi_L^{+}\Psi_R$ are Lorentz invariant.

Let us now put

(9) $\quad \theta = \Psi_R^{+} \Psi_L + \bar{\Psi}_R \bar{\Psi}_L^{+}$, This is not a hypercharge eigenstate.

θ is Lorentz invariant and it obeys $\theta = \bar{\theta}^{+}$ so that it has the form:

$$\theta = \begin{pmatrix} \theta^0 & -\theta^- \\ \theta^+ & \bar{\theta}^0 \end{pmatrix} = \Theta + \bar{\Theta}^{+} \quad \text{where } \Theta = \theta \frac{1+\sigma_3}{2} = \begin{pmatrix} \theta^0 & 0 \\ \theta^+ & 0 \end{pmatrix}$$

Also define $\theta_1^0 = \frac{\theta + \bar{\theta}}{2}$ $\vec{\sigma}.\vec{\theta} = \frac{\theta - \bar{\theta}}{2}$.

the transformation law is

$$\theta \rightarrow \bar{R}\,\theta\, Q .$$

We now form

$$\vec{\phi} = \theta \sigma_3 \bar{\theta} \qquad \phi_0 = \theta \bar{\theta}$$

this is not Q invariant but it is invariant against $Q = e^{\xi u}$.

We now have the transformations:

isotopic spin transf. $\begin{cases} \Psi_L \rightarrow \Psi_L R \\ \Psi_R \rightarrow \Psi_R \end{cases}$ $\quad \theta \rightarrow \bar{R}\theta$ or $\begin{cases} \Theta \rightarrow \bar{R}\theta \\ \bar{\Theta}^{+} \rightarrow \bar{R}\,\bar{\Theta}^{+} \end{cases}$ $\quad \vec{\phi} \rightarrow \bar{R}\,\phi\, R .$

hypercharge gauge transf. $\begin{cases} \Psi_L \rightarrow \Psi_L e^{i\sigma_3 u} \\ \Psi_R \rightarrow \Psi_R e^{-i\sigma_3 u} \end{cases}$ $\quad \theta \rightarrow \theta e^{i\sigma_3 u}$ or $\begin{cases} \Theta \rightarrow \Theta e^{iu} \\ \bar{\Theta}^{+} \rightarrow \Theta e^{-iu} \end{cases}$ $\quad \vec{\phi} \rightarrow \vec{\phi}$

in particular if $u = \frac{\pi}{2}$

Racah's parity $\begin{cases} \Psi_L \rightarrow i\Psi_L \\ \Psi_R \rightarrow -i\Psi_R \end{cases}$ $\quad \theta \rightarrow \theta i\sigma_3$ or $\begin{cases} \Theta \rightarrow i\Theta \\ \bar{\Theta}^{+} \rightarrow -i\Theta \end{cases}$ $\quad \phi \rightarrow \phi.$

rotation in charge space $\begin{cases} \Psi_L \rightarrow \Psi_L M \\ \Psi_R \rightarrow \Psi_R M \end{cases}$ $\quad \theta \rightarrow \bar{M}\theta M$ $\begin{cases} \theta_1^0 \rightarrow \theta_1^0 \\ \vec{\sigma}.\vec{\theta} \rightarrow \bar{M}(\vec{\sigma}.\vec{\theta})M . \end{cases}$ $\quad \cancel{\phi \rightarrow \bar{M}\phi M}$: ϕ ~~is not an eigenstate~~ transforms like a tensor.

special rotation in charge space $\begin{cases} \Psi_L \rightarrow \Psi_L e^{i\sigma_3 t_3} \\ \Psi_R \rightarrow \Psi_R e^{i\sigma_3 \xi} \end{cases}$ $\quad \theta \rightarrow e^{-\xi t_3}\theta e^{\xi t_3}$ $\quad \phi \rightarrow e^{-\xi t_3}\phi e^{\xi t_3}$