1) Contribution of $T=\frac{3}{2}$ state   Current ... of the ...

$$\varepsilon = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \qquad \frac{1-\varepsilon}{2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2} \cdot \begin{pmatrix} n^0 & \sqrt{2}n^+ & & \\ \sqrt{2}n^- & -n^0 & & \\ & & n^0 & \sqrt{2}n^+ \\ & & \sqrt{2}n^- & -n^0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2}\sqrt{2}n^+ & -\sqrt{2}n^+ & 0 \\ 0 & -\frac{n^0}{2} & \frac{n^0}{2} & 0 \\ 0 & -\frac{n^0}{2} & \frac{n^0}{2} & 0 \\ 0 & -\frac{\sqrt{2}n^-}{2} & \frac{\sqrt{2}n^-}{2} & \end{pmatrix}$$

$$\varepsilon = \frac{1+\vec{\rho}\cdot\vec{\sigma}}{2} \qquad \varepsilon^2 = 1.$$

$$\Pi = \vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2} = -\vec{\rho}\cdot\vec{n}\,\frac{1-\varepsilon}{2}$$

$$\vec{\sigma}\cdot\vec{n} \to e^{\frac{i}{2}\vec{\sigma}\cdot\vec{\omega}}\,\vec{\sigma}\cdot\vec{n}\,e^{-\frac{i}{2}\vec{\sigma}\cdot\vec{\omega}}$$

$$\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2} \to e^{i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}\,\vec{\sigma}\cdot\vec{n}\,e^{-i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}\,\frac{1-\varepsilon}{2} = e^{i\sigma\cdot\frac{\omega}{2}}\,\sigma\cdot n\,e^{i\vec{\rho}\cdot\frac{\vec{\omega}}{2}}\,\frac{1-\varepsilon}{2}$$

$$\varepsilon\,\Pi = -\vec{\rho}\cdot\vec{n}\,\frac{1-\varepsilon}{2} \qquad \underline{\Pi \to e^{i(\sigma+\rho)\cdot\frac{\vec{\omega}}{2}}\,\Pi}$$

$$\Pi = \varepsilon\,\Pi = \frac{1+\varepsilon}{2}\Pi = \vec{\sigma}\cdot\vec{n}\cdot\frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2} = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}n^+ & -\sqrt{2}n^+ & 0 \\ 0 & -n^0 & n^0 & 0 \\ 0 & -n^0 & n^0 & 0 \\ 0 & -\sqrt{2}n^- & \sqrt{2}n^- & 0 \end{pmatrix}$$

or

$$\Pi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & n^+ & -n^+ & 0 \\ 0 & -\frac{n^0}{\sqrt{2}} & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & -\frac{n^0}{\sqrt{2}} & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & -n^- & n^- & 0 \end{pmatrix} = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2}$$

$$\begin{pmatrix} n^- & -\frac{n^0}{\sqrt{2}} & -\frac{n^0}{\sqrt{2}} & -n^+ \end{pmatrix}\begin{pmatrix} n^+ \\ -\frac{n^0}{\sqrt{2}} \\ -\frac{n^0}{\sqrt{2}} \\ -n^- \end{pmatrix} = 2\left(n^-n^+ + \frac{n^{0\,2}}{2}\right) = 2n^-n^+ + n^{0\,2}$$

$$\Pi \to e^{i(\rho+\sigma)\frac{\vec{\omega}}{2}}\,\Pi \qquad \begin{pmatrix} p \\ n \end{pmatrix} \to e^{i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}\begin{pmatrix} p \\ n \end{pmatrix} \qquad (-n\ \ p) \to (-n\ \ p)\,e^{-i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}$$

$$\begin{pmatrix} -n & p \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} -n & p \\ 0 & 0 \end{pmatrix} e^{-i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}$$

$$N' = \begin{pmatrix} -n & p & & \\ 0 & 0 & & \\ & & -n & p \\ & & 0 & 0 \end{pmatrix} \qquad \frac{1+\sigma_3}{2}\begin{pmatrix} -n & p \\ p & n \end{pmatrix} \qquad N = e^{i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}\begin{pmatrix} 0 & p & & \\ 0 & n & & \\ & & 0 & p \\ & & 0 & n \end{pmatrix}$$

$$N \to e^{-i\sigma\cdot\frac{\omega}{2}}$$

$$N' = \sigma_2 N^T \sigma_2$$

$$\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2}\,\frac{1-\sigma_3}{2}(-n+\sigma_1 p)$$

$$\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2}\,\frac{1-\sigma_3}{2}\,\frac{1+\sigma_3}{2}$$

$$\Pi N' \to e^{i(\rho+\sigma)\frac{\vec{\omega}}{2}}\,\Pi N' e^{-i\sigma\cdot\frac{\omega}{2}}$$

$\rho_1 - \sigma_1$

$$\frac{1-\rho_1\sigma_1-\rho_2\sigma_2-\rho_3\sigma_3}{2}\,\rho_1$$

$$\frac{1-\rho_1\sigma_1-\rho_2\sigma_2-\rho_3\sigma_3}{2}\,\sigma_1$$

$$N\,\vec{\rho}\cdot\vec{n} \to e^{i\sigma\frac{\omega}{2}}e^{i\rho\cdot\frac{\omega}{2}}\,N\,\vec{\rho}\cdot\vec{n}\,e^{-i\sigma\cdot\frac{\omega}{2}}$$

$$\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2}\left(p\frac{\sigma^+}{\sqrt{2}} - n\frac{1+\sigma_3}{2}\right) = \Pi N' = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & n^+ & -n^+ & 0 \\ 0 & -\frac{n^0}{\sqrt{2}} & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & -\frac{n^0}{\sqrt{2}} & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & -n^- & n^- & 0 \end{pmatrix}\begin{pmatrix} -n & p & & \\ 0 & 0 & & \\ & & -n & p \\ & & 0 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & nn^+ & -pn^+ \\ 0 & 0 & -\frac{nn^0}{\sqrt{2}} & p\frac{n^0}{\sqrt{2}} \\ 0 & 0 & -\frac{nn^0}{\sqrt{2}} & p\frac{n^0}{\sqrt{2}} \\ 0 & 0 & -nn^- & pn^- \end{pmatrix}$$

$$= \vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2}\left(\frac{p\sigma^+}{\sqrt{2}} - n\frac{1+\sigma_3}{2}\right)$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{\sigma_1+i\sigma_2}{2} = \frac{1}{\sqrt{2}}\sigma^+$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1-\sigma_3}{2} = \frac{1}{\sqrt{2}}\,\frac{1-\sigma_3}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -n^+ & 0 \\ 0 & 0 & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & 0 & n^- & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -n & p \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\frac{9}{2}$? $= \frac{3}{2}$ stats

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$\sigma^+ = \frac{\sigma_1 + i\sigma_2}{\sqrt{2}}$ $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$$N_? = \begin{pmatrix} 0 & p & & \\ 0 & n & & \\ & & 0 & p \\ & & 0 & n \end{pmatrix} = \frac{\sigma^+}{\sqrt{2}} p + n \frac{1-\sigma_3}{2} \quad \to \quad e^{i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}} N .$$

$$\frac{1+\varepsilon}{2} N = \begin{pmatrix} 1 & & & \\ & \frac{1}{2} & \frac{1}{2} & \\ & \frac{1}{2} & \frac{1}{2} & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 0 & p & & \\ 0 & n & & \\ & & 0 & p \\ & & 0 & n \end{pmatrix} = \begin{pmatrix} 0 & p & 0 & 0 \\ 0 & \frac{n}{2} & 0 & \frac{p}{2} \\ 0 & \frac{n}{2} & 0 & \frac{p}{2} \\ 0 & 0 & 0 & n \end{pmatrix}$$

$$N_\rho = \varepsilon N_\sigma \varepsilon = \frac{\rho^+}{\sqrt{2}} p + n \frac{1-\rho_3}{2} = \begin{pmatrix} 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{pmatrix} \qquad \frac{1+\varepsilon}{2} N_\rho = \begin{pmatrix} 0 & 0 & p & 0 \\ 0 & 0 & \frac{n}{2} & \frac{p}{2} \\ 0 & 0 & \frac{n}{2} & \frac{p}{2} \\ 0 & 0 & 0 & n \end{pmatrix}$$

$$\frac{1+\varepsilon}{2} N_\rho \, \vec{\sigma}\cdot\vec{\pi} \quad \to \quad e^{i(\rho+\sigma)\cdot\frac{\omega}{2}} \left(\frac{1+\varepsilon}{2} N_\rho \vec{\sigma}\cdot\vec{\pi} \right) e^{-i\sigma\frac{\omega}{2}}$$

$\frac{1+\varepsilon}{2} \left(\frac{\rho^+}{\sqrt{2}} p + n \frac{1-\rho_3}{2} \right) \vec{\sigma}\cdot\vec{\pi}$ and $\vec{\sigma}\cdot\vec{\pi} \frac{1-\varepsilon}{2} \left(\frac{p\rho^+}{\sqrt{2}} + n \frac{1-\rho_3}{2} \right)$ transform in the same way.

$$\vec{\sigma}\cdot\vec{\pi} \frac{1-\varepsilon}{2} \left(\frac{p\rho^+}{\sqrt{2}} + n\frac{1-\rho_3}{2} \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -n\pi^+ & p\pi^+ \\ 0 & 0 & n\frac{\pi^0}{\sqrt{2}} & -p\frac{\pi^0}{\sqrt{2}} \\ 0 & 0 & n\frac{\pi^0}{\sqrt{2}} & -p\frac{\pi^0}{\sqrt{2}} \\ 0 & 0 & n\pi^- & -p\pi^- \end{pmatrix}$$

$$\frac{1+\varepsilon}{2} \left(\frac{p\rho^+}{\sqrt{2}} + n\frac{1-\rho_3}{2} \right) \vec{\sigma}\cdot\vec{\pi} = \begin{pmatrix} 0 & 0 & p & 0 \\ 0 & 0 & \frac{n}{2} & \frac{p}{2} \\ 0 & 0 & \frac{n}{2} & \frac{p}{2} \\ 0 & 0 & 0 & n \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \pi^0 & \sqrt{2}\pi^+ \\ 0 & 0 & \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & p\pi^0 & \sqrt{2}p\pi^+ \\ 0 & 0 & \frac{n}{2}\pi^0 & \frac{\sqrt{2}}{2}n\pi^+ - \frac{p\pi^0}{2} \\ 0 & 0 & \frac{n}{2}\pi^0 & \frac{\sqrt{2}}{2}n\pi^+ - \frac{p\pi^0}{2} \\ 0 & 0 & \sqrt{2}n\pi^- & -n\pi^0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{3}}T_{+\frac{1}{2}} & T_{\frac{3}{2}} \\ \frac{1}{\sqrt{3}}T_{-\frac{1}{2}} & -\frac{1}{\sqrt{3}}T_{\frac{1}{2}} \\ \frac{1}{\sqrt{3}}T_{-\frac{1}{2}} & -\frac{1}{\sqrt{3}}T_{\frac{1}{2}} \\ T_{-\frac{3}{2}} & -\frac{1}{\sqrt{3}}T_{-\frac{1}{2}} \end{pmatrix} e^{i(\vec{\tau}+\vec{\sigma})\cdot\frac{\vec{\omega}}{2}} = \begin{pmatrix} \frac{1}{\sqrt{3}}n & p \\ \frac{1}{\sqrt{3}}n^{x} & -\frac{1}{\sqrt{3}}n \\ \frac{1}{\sqrt{3}}n^{x} & -\frac{1}{\sqrt{3}}n \\ p^{x} & -\frac{1}{\sqrt{3}}n^{x} \end{pmatrix} e^{-i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}$$

$$\omega_3 = \frac{N}{2} + Q$$

$$Q = I_3 + \frac{N-S}{2} = I_3 + \frac{N}{2} - \frac{S}{2} = I_3 + \frac{U}{2}$$

when $S=0$ $\quad \omega_3 = Q + \frac{N}{2} = I_3 + N$

$$\omega_3 = Q + \frac{N}{2} = I_3 + N - \frac{S}{2} = I_3 + \frac{N}{2} + \frac{U}{2}.$$

$S=-2$

for $=0$ $\quad \omega_3 = +\frac{1}{2} + 1 + 1$

when $U=-1$ $\quad \omega_3 = I_3 + \frac{N}{2} = 1$

3) $T = \frac{3}{2}$ states

$\mathcal{D}_+$

$$\vec{\sigma}\cdot\vec{\pi}\,\frac{1-\varepsilon}{2}\left(\frac{p\rho^+}{\sqrt{2}} + n\,\frac{1-\rho_3}{2}\right) = \begin{pmatrix} 0 & 0 & -n\frac{\pi^+}{\sqrt{2}} & \frac{1}{\sqrt{2}}p\pi^+ \\ 0 & 0 & n\frac{\pi^0}{2} & -p\frac{\pi^0}{2} \\ 0 & 0 & n\frac{\pi^0}{2} & -p\frac{\pi^0}{2} \\ 0 & 0 & n\frac{\pi^-}{\sqrt{2}} & -p\frac{\pi^-}{\sqrt{2}} \end{pmatrix}$$

$$\vec{\sigma}\cdot\vec{\pi}\cdot\frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{\pi}\,\frac{1-\varepsilon}{2}$$

$$\frac{1+\varepsilon}{2}\left(\frac{p\rho^+}{\sqrt{2}} + n\,\frac{1-\rho_3}{2}\right)\vec{\sigma}\cdot\vec{\pi} = \begin{pmatrix} 0 & 0 & p\pi^0 & \frac{2}{\sqrt{2}}p\pi^+ \\ 0 & 0 & \frac{n}{2}\pi^0 + p\frac{\pi^-}{\sqrt{2}} & n\frac{\pi^+}{\sqrt{2}} - p\frac{\pi^0}{2} \\ 0 & 0 & n\frac{\pi^0}{2} + p\frac{\pi^-}{\sqrt{2}} & n\frac{\pi^+}{\sqrt{2}} - p\frac{\pi^0}{2} \\ 0 & 0 & \frac{2}{\sqrt{2}}n\pi^- & -n\pi^0 \end{pmatrix}$$

$$T = \vec{\sigma}\cdot\vec{\pi}\,\frac{1-\varepsilon}{2}\left(\frac{p\rho^+}{\sqrt{2}} + n\,\frac{1-\rho_3}{2}\right) + \frac{1+\varepsilon}{2}\left(\frac{p\rho^+}{\sqrt{2}} + n\,\frac{1-\rho_3}{2}\right)\vec{\sigma}\cdot\vec{\pi} = \begin{pmatrix} 0 & 0 & p\pi^0 - n\frac{\pi^+}{\sqrt{2}} & \frac{3}{\sqrt{2}}p\pi^+ \\ 0 & 0 & n\pi^0 + p\frac{\pi^-}{\sqrt{2}} & -p\pi^0 + n\frac{\pi^+}{\sqrt{2}} \\ 0 & 0 & n\pi^0 + p\frac{\pi^-}{\sqrt{2}} & -p\pi^0 + n\frac{\pi^+}{\sqrt{2}} \\ 0 & 0 & \frac{3}{\sqrt{2}}n\pi^- & -n\pi^0 - p\frac{\pi^-}{\sqrt{2}} \end{pmatrix}$$

$$T = \frac{3}{\sqrt{2}}\begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{3}}T_{+\frac{1}{2}} & T_{+\frac{3}{2}} \\ 0 & 0 & \frac{1}{\sqrt{3}}T_{-\frac{1}{2}} & -\frac{1}{\sqrt{3}}T_{+\frac{1}{2}} \\ 0 & 0 & \frac{1}{\sqrt{3}}T_{-\frac{1}{2}} & -\frac{1}{\sqrt{3}}T_{+\frac{1}{2}} \\ 0 & 0 & T_{-\frac{3}{2}} & -\frac{1}{\sqrt{3}}T_{-\frac{1}{2}} \end{pmatrix}$$

$$T \to e^{i(\vec{\rho}+\vec{\sigma})\cdot\frac{\vec{\omega}}{2}}\,T\,e^{-i\frac{\vec{\omega}}{2}}$$

$$T = \frac{1+\varepsilon}{2}\left\{\vec{\sigma}\cdot\vec{\pi}\,\frac{1-\varepsilon}{2}\,(p\rho_1 + n) + (p\rho_1 + n)\,\vec{\sigma}\cdot\vec{\pi}\right\}\frac{1-\rho_3}{2}$$

$$= \frac{1+\varepsilon}{2}\left\{\frac{3}{2}(\vec{\sigma}\cdot\vec{\pi})(p\rho_1 + n) - \frac{1}{2}(\vec{\sigma}\cdot\vec{\pi})\cdot\varepsilon\,(p\rho_1 + n)\right\}\frac{1-\rho_3}{2} = \frac{1+\varepsilon}{2}\left\{\left(\frac{3}{2}\vec{\sigma}\cdot\vec{\pi} - \frac{i}{2}\vec{\rho}\cdot\vec{\pi}\right)(p\rho_1 + n)\right\}\frac{1-\rho_3}{2}$$

$$= \frac{1+\varepsilon}{2}\left\{\left(\vec{\sigma} + \frac{\vec{\sigma}-\vec{\rho}}{2}\right)\cdot\vec{\pi}\left(p\,\frac{\rho_1 + i\rho_2}{2} + n\,\frac{1-\rho_3}{2}\right)\right\}$$

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$\frac{i}{2}\vec{\rho}\times\vec{\sigma}\cdot\vec{a} =$

$9\sigma + 3\rho - 3i\rho\times\sigma \quad -3\rho - \sigma - i\rho\cdot\sigma$

$\frac{1+e}{2}\vec{\sigma}$

$$\frac{1+e}{2}\vec{\sigma} = \frac{3\vec{\sigma}+\vec{\rho} \pm i\vec{\rho}\times\vec{\sigma}}{4}$$

$$\frac{1}{2}\left(1 + \frac{1+\rho\cdot\sigma}{2}\right)\Big|_{\sigma_1} = \frac{3+\rho_1\sigma_1+\rho_2\sigma_2+\rho_3\sigma_3}{4}\,\sigma_1 = \frac{3\sigma_1+\rho_1+i(\rho_3\sigma_2-\rho_2\sigma_3)}{4}$$

$3\rho_1+\sigma_1 + i(\rho_2\sigma_3 - \rho_3\sigma_2)$

$$\frac{1+e}{2}\vec{\rho} = \frac{3\vec{\rho}+\vec{\sigma}+i\vec{\rho}\times\vec{\sigma}}{4}$$

$$\frac{1+e}{2}\,\frac{3\vec{\sigma}-\vec{\rho}}{2} = \frac{8\vec{\sigma}-4i\vec{\rho}\times\vec{\sigma}}{8} = \vec{\sigma}-\frac{1}{2}i\vec{\rho}\times\vec{\sigma}$$

$$\sigma_1 - i\,\frac{\rho_2\sigma_3-\rho_3\sigma_2}{2}$$

4)

$$T = \frac{1+\varepsilon}{2}\left\{\left(\vec\sigma\cdot\vec n + \frac{\vec\sigma-\vec\rho}{2}\cdot\vec n\right)(p\rho_1+m)\right\}\frac{1-\rho_3}{2}$$

$$\frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec n = \frac{\vec\sigma\cdot\vec n}{2} + \varepsilon\frac{\vec\sigma\cdot\vec n}{2} = \frac{\vec\sigma\cdot\vec n}{2} + \frac{\vec\rho\cdot\vec n}{2}\varepsilon$$

$$\frac{1+\varepsilon}{2}\,\vec\rho\cdot\vec n = \frac{\vec\rho\cdot\vec n}{2} + \frac{\vec\sigma\cdot\vec n}{2}\varepsilon$$

Identities:

$$\frac{1+\varepsilon}{2}\,\frac{\vec\sigma-\vec\rho}{2}\cdot\vec n = \frac{\vec\sigma\cdot\vec n}{2}\,\frac{1-\varepsilon}{2} - \frac{\vec\rho\cdot\vec n}{2}\,\frac{1-\varepsilon}{2} = \left(\frac{\vec\sigma-\vec\rho}{2}\right)\cdot\vec n\;\frac{1-\varepsilon}{2}$$

$$\frac{1+\varepsilon}{2}\,\frac{\vec\sigma-\vec\rho}{2} = \frac{\vec\sigma-\vec\rho}{2}\,\frac{1-\varepsilon}{2}.\quad \text{because}\quad \varepsilon(\sigma-\rho) = -(\sigma-\rho)\varepsilon$$

$$\frac{1-\varepsilon}{2}\,\frac{\vec\sigma+\vec\rho}{2} = 0.$$

$$\frac{\sigma+\rho}{2}\,\frac{1-\varepsilon}{2} = 0$$

thus $\boxed{\vec\sigma\,\frac{1-\varepsilon}{2} = -\vec\rho\,\frac{1-\varepsilon}{2} = \frac{\vec\sigma-\vec\rho}{2}\,\frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2}\,\frac{\vec\sigma-\vec\rho}{2}.}$

$$\vec\sigma\cdot\vec n\,\frac{1-\varepsilon}{2} = -\vec\rho\cdot\vec n\,\frac{1-\varepsilon}{2} = \frac{(\vec\sigma-\vec\rho)}{2}\cdot\vec n\,\frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2}\,\frac{\vec\sigma-\vec\rho}{2}\cdot\vec n.$$

$$T = \frac{1+\varepsilon}{2}\left\{\vec\sigma\cdot\vec n\,\frac{1-\varepsilon}{2}\,(p\rho_1+m) + \vec\sigma\cdot\vec n\,(p\rho_1+m)\right\}\frac{1-\rho_3}{2}$$

$$T = \frac{1+\varepsilon}{2}\left\{\frac{\vec\sigma-\vec\rho}{2}\cdot\vec n\,(p\rho_1+m) + \vec\sigma\cdot\vec n\,(p\rho_1+m)\right\}\frac{1-\rho_3}{2}$$

$$\frac{3\vec\sigma-\vec\rho}{2}$$

$$= \frac{1+\varepsilon}{2}\left\{\left(\vec\sigma + \frac{\vec\sigma-\vec\rho}{2}\right)\cdot\vec n\,(p\rho_1+m)\,\frac{1-\rho_3}{2}\right)$$

$$T' = \frac{2\sqrt2}{3}T \quad \frac{1+\varepsilon}{\sqrt2}\Big\{ \qquad T^{(\frac{3}{2})} = \frac{1+\varepsilon}{2}\left\{\left(\frac{3}{2}\vec\sigma - \frac{\vec\rho}{2}\right)\cdot\vec n\,(p\rho_1+m)\,\frac{1-\rho_3}{2}\right\}$$

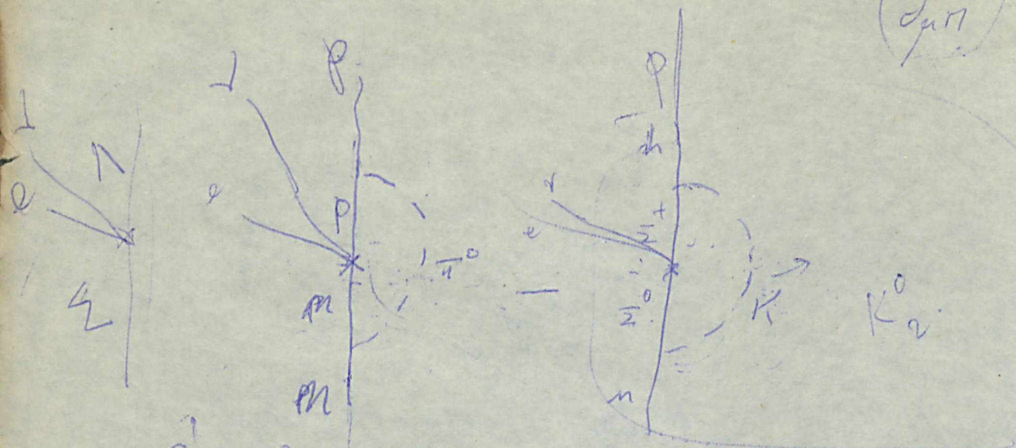
$$= \frac{1+\varepsilon}{2}\left\{\frac{3}{2}\,\vec\sigma\cdot\vec n\,(p\rho_1+m) - \frac{1}{2}(\vec\rho\cdot\vec n)(p\rho_1+m)\right\}\frac{1-\rho_3}{2}.$$

$$= \left(\vec\sigma - i\tfrac{1}{2}\vec\rho\times\vec\sigma\right)\cdot\vec n\,(p\rho_1+m)\,\frac{1-\rho_3}{2}$$

$(\partial_\mu \vec{\pi})$

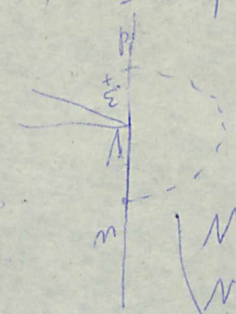
$\bar{p}\gamma_\mu n + \bar{\Xi}^0\gamma_\mu \Xi^-$

$\bar{p}\gamma_{5\mu} n - \bar{\Xi}^0\gamma_{5\mu}\Xi^-$



$K = K_1^0 + \vec{\tau}\cdot\vec{K}$

K_2^0



$\Xi \to N^G$

$N \to e^{i\vec{\tau}\cdot\vec{\omega}} N$

$N^G \to e^{i\tau\omega} N^G$

$\Xi \to e^{i\tau\omega}\Xi$

$G_A^1 = G_V$

$G_A^1 = G_V^1$

$$\frac{(m_K+m_\pi)(m_K-m_\pi)}{m_B^2} = \frac{8\, m_\pi^2}{50\, m_\pi^2} \sim 16\%$$

$\partial_\mu \bar{\psi}\gamma_{5\mu}\psi = 0$

$N \to e^{i\gamma_5\vec{\tau}\cdot\vec{\omega}'}N$

$N^G \to e^{-i\gamma_5\vec{\tau}\cdot\vec{\omega}'}N^G$

$\Xi \to e^{-i\gamma_5}$

$(\vec{K} \to \vec{\pi})$ $\quad g \quad N_1 N_2$

$g_1\partial_\mu\vec{\pi} + g_1'\partial_\mu\vec{K} + \bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi) = 0$

$(\vec{\pi}, \vec{K})$ $\quad \vec{\pi} \rightleftarrows \vec{K}$

$\mu_\pi^2\pi$

$\mu^2 K$

$S = G S_\mu + G' S_\mu'$

$\gamma_\mu \quad \gamma_5\gamma_\mu$

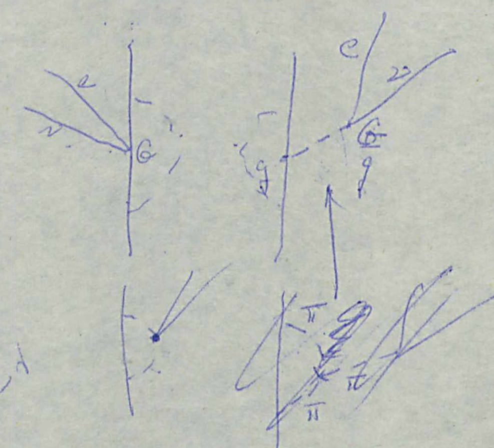
$\Box e^{\pi} = \lambda^2 e^{\pi}$

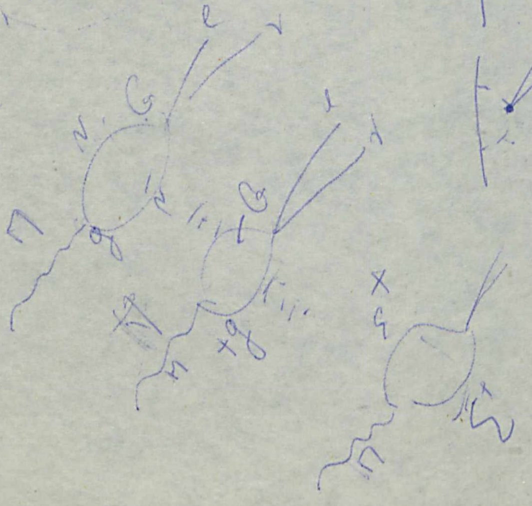
$e^{\pi} \to e$

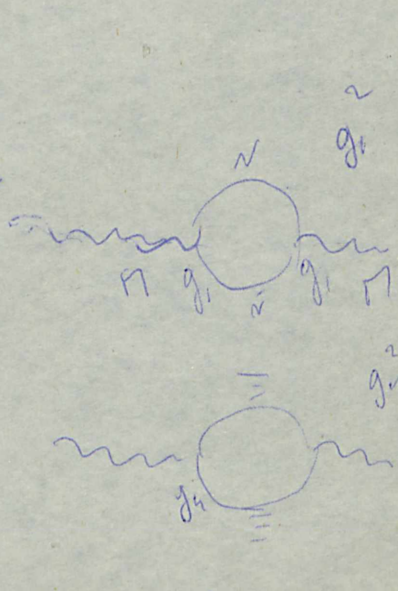
$I = I + K$

$K(K+1) \quad K_3$

$I(I+1) \quad I_3$







$T = \frac{1}{2}$ amplitude.

$$\vec{\rho}\cdot\vec{\pi}\cdot(p\rho_1+n)\frac{1-\rho_3}{2} = \begin{pmatrix} \pi_0 & 0 & \sqrt{2}\pi^+ & 0 \\ 0 & +\pi_0 & 0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & 0 & -\pi_0 & 0 \\ 0 & \sqrt{2}\pi^- & 0 & -\pi_0 \end{pmatrix} \begin{pmatrix} 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & p\pi_0+\sqrt{2}n\pi^+ & 0 \\ 0 & 0 & 0 & p\pi_0+\sqrt{2}n\pi^+ \\ 0 & 0 & \sqrt{2}p\pi^- - n\pi_0 & 0 \\ 0 & 0 & 0 & \sqrt{2}p\pi^- - n\pi_0 \end{pmatrix}$$

$(\vec{\sigma}\cdot\vec{\pi})(p\rho_1+n)\frac{1-\rho_3}{2}$ contains both $T=\frac{3}{2}$ and $T=\frac{1}{2}$ amplitudes. (.6 s late).

Subtracting the $T=\frac{1}{2}$ amplitude we get the $T=\frac{3}{2}$ amplitude.

n $\qquad \Sigma^+ p\pi_0$ contains both $T=\frac{3}{2}$ and $\frac{1}{2}$ amplitudes.

The total amplitude is

$$O_1 \quad \overline{\Sigma^+} O_1 (p\pi_0+\sqrt{2}n\pi^+) - \frac{\overline{\Sigma^0}}{\sqrt{2}} O_1 (p\pi_0+\sqrt{2}n\pi^+)$$

$$+ \overline{\Sigma^+} O_2 \left(p\pi^0 - n\frac{\pi^+}{\sqrt{2}}\right) - 2\frac{\overline{\Sigma^0}}{\sqrt{2}} O_2 \left(n\pi^0 + p\frac{\pi^-}{\sqrt{2}}\right) - \frac{3}{\sqrt{2}}\overline{\Sigma^-} O_2\, n\pi^- .$$

$$= \overline{\Sigma^+}(O_1+O_2)p\pi^0 + \overline{\Sigma^+}\left(O_1\sqrt{2} - \frac{O_2}{\sqrt{2}}\right)n\pi^+ - \frac{3}{\sqrt{2}}\overline{\Sigma^-} O_2\, n\pi^- + .$$

experimentally $O_1 = 1$, $O_2 = \gamma_5$?

then charged mode: $\sqrt{2}\,\overline{\Sigma^+}\cdot(1-\frac{1}{2}\gamma_5)\,n\pi^+$

(6)

Σ decay, $\to N + \pi$

a) $\Sigma^+ \to p + \pi^0$, b) $\Sigma^+ \to n + \pi^+$

c) $\Sigma^- \to n + \pi^-$ d) $\Sigma^0 \to p + \pi^-$ e) $\Sigma^0 \to n + \pi^0$.

There are 5 amplitudes. In the Σ decay of the charged Σ's there are 3 amplitudes. Since parity is not conserved there are 6 amplitudes.

c) is a pure $T = \frac{3}{2}$ state in the final state. a and b are mixtures of $T = \frac{1}{2}$ and $T = \frac{3}{2}$.

$n\pi^-$ T:

Transform like $e^{i\alpha}e^{i\tau\frac{\alpha}{2}}(\ )e^{-i\tau\frac{\alpha}{2}}$

$$\begin{pmatrix} \sqrt{2}\left(n\pi^0 + p\frac{\pi^-}{\sqrt{2}}\right) & \sqrt{2}\left(p\pi^0 - n\frac{\pi^+}{\sqrt{2}}\right) \\ -3n\pi^- & -\sqrt{2}\left(n\pi^0 + p\frac{\pi^-}{\sqrt{2}}\right) \end{pmatrix} \cdot \begin{pmatrix} \bar{\Sigma}^0 & \sqrt{2}\,\bar{\Sigma}^- \\ \sqrt{2}\,\bar{\Sigma}^+ & -\bar{\Sigma}^0 \end{pmatrix}$$

$T = \frac{3}{2}$

$$= \sqrt{2}\, O_1 \left(\bar{\Sigma}^0 (\sqrt{2}\, n\pi^0 + p\pi^-) + 2\bar{\Sigma}^+ \left(p\pi^0 - n\frac{\pi^+}{\sqrt{2}}\right) - 3\sqrt{2}\,\bar{\Sigma}^- n\pi^- + \bar{\Sigma}^0 (\sqrt{2}\, n\pi^0 + p\pi^-) \right]$$

$$\begin{pmatrix} n\pi_0 - \sqrt{2}\,p\pi^- & p\pi_0 + \sqrt{2}\,n\pi^+ \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{\Sigma}^0 & \sqrt{2}\,\bar{\Sigma}^- \\ \sqrt{2}\,\bar{\Sigma}^+ & -\bar{\Sigma}^0 \end{pmatrix} = \begin{pmatrix} \bar{\Sigma}^0 (n\pi_0 - \sqrt{2}\,p\pi^-) + \sqrt{2}\,\bar{\Sigma}^+ (p\pi_0 + \sqrt{2}\,n\pi^+) & (\ ) \\ 0 & 0 \end{pmatrix}$$

$2O_2 -$

$$\begin{pmatrix} 0 & 0 \\ p\pi_0 + \sqrt{2}\,n\pi^+ & \end{pmatrix}$$

$$\sqrt{2}\,\bar{\Sigma}^+ (\sqrt{2}\,O_2 - O_1) + \sqrt{2}\,\bar{\Sigma}^+ (\sqrt{2}\,O_1 + O_2)\,p\pi^0 - 3\sqrt{2}\,\bar{\Sigma}^- O_1 n\pi^-$$

$$O_1 \left(2\bar{\Sigma}^+ \left(p\pi^0 - n\frac{\pi^+}{\sqrt{2}}\right) - 3\sqrt{2}\,\bar{\Sigma}^- n\pi^- + \bar{\Sigma}^0 (\ ) \right) + O_2 \left(\sqrt{2}\,\bar{\Sigma}^+ (p\pi_0 + \sqrt{2}\,n\pi^+ + \bar{\Sigma}^0 (\ ) \right)$$

$(a + b\gamma_5)$

$$\bar{\Sigma}^+ \left(2O_2 - \frac{\sqrt{2}}{\cancel{\sqrt{2}}} O_1\right) n\pi^+ + \bar{\Sigma}^+ (2O_1 + \sqrt{2}\,O_2)\,p\pi^0 - 3\sqrt{2}\,\bar{\Sigma}^- O_1 n\pi^-$$

$$\left(T^1_{-1} \quad -\tfrac{1}{\sqrt2} T^1_0 \quad -\tfrac{1}{\sqrt2} T^1_0 \quad -T^1_1 \right) \begin{pmatrix} \frac{1}{\sqrt3} T^{\frac32}_{+\frac12} \\ \frac{1}{\sqrt3} T^{\frac32}_{-\frac12} \\ \frac{1}{\sqrt3} T^{3/2}_{-\frac12} \\ T^{\frac32}_{-\frac32} \end{pmatrix} = T^{\frac12}_{-\frac12}$$

$$\left(T^1_{-1} \quad -\tfrac{1}{\sqrt2} T^1_0 \quad -\tfrac{1}{\sqrt2} T^1_0 \quad -T^1_1 \right) \begin{pmatrix} T^{\frac32}_{+\frac32} \\ -\frac{1}{\sqrt3} T^{\frac32}_{+\frac12} \\ -\frac{1}{\sqrt3} T^{\frac32}_{+\frac12} \\ -\frac{1}{\sqrt3} T^{\frac32}_{-\frac12} \end{pmatrix} = T^{\frac12}_{+\frac12}$$

$M = m + m'$

$$T^J_M = \sum T^j_m T^{j'}_{m'} C^{j j' J}_{m m' M}$$

$$T^{\frac12}_{-\frac12} = \frac{1}{\sqrt3} T^{\frac32}_{+\frac12} T^1_{-1} - \sqrt{\frac23}\, T^{\frac32}_{-\frac12} T^1_0 - T^{\frac32}_{-\frac32} T^1_{+1}$$

$$T^{\frac12}_{+\frac12} = T^{\frac32}_{+\frac32} T^1_{-1} + \sqrt{\frac23}\, T^{\frac32}_{+\frac12} T^1_0 + \frac{1}{\sqrt3} T^{\frac32}_{-\frac12} T^1_{+1}$$

$$C^{\frac32\ 1\ \frac12}_{\frac32\ -1\ \frac12}$$

$j = 1$

$M = -1$

$$(-1)^{2-\frac12} \sqrt{\frac{1+1+\frac12}{(2+2)(2+1)}} =$$

$= \sqrt{}$

$$\begin{pmatrix} T^{\frac12}_{+\frac12} \\ T^{\frac12}_{-\frac12} \end{pmatrix} = ($$

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$1 - \varepsilon + 2$

$$= \frac{1+\varepsilon}{2}\left\{\vec{\sigma}\cdot\vec{n}\ \left(\frac{1-\varepsilon}{2} N + N\right)\right\}$$

$\frac{N}{2} + \frac{N}{4} - \varepsilon\frac{N}{2} \quad 3N -$

$$= \frac{1+\varepsilon}{2}\left\{\vec{\sigma}\cdot\vec{n}\ \ast\left(\frac{3-\varepsilon}{2} N\right)\right\}$$

$\frac{3}{2} - \frac{\varepsilon}{2}$

$$\frac{\varepsilon}{2} = \begin{pmatrix} \frac{1}{2} & & & \\ & & \frac{1}{2} & \\ & \frac{1}{2} & & \\ & & & \frac{1}{2} \end{pmatrix} \qquad \frac{3}{2}\begin{pmatrix} \frac{3}{2} & & & \\ & \frac{3}{2} & & \\ & & \frac{3}{2} & \\ & & & \frac{3}{2} \end{pmatrix}$$

$$\frac{3-\varepsilon}{2} = \begin{pmatrix} 2 & & & \\ & \frac{3}{2} & \frac{1}{2} & \\ & \frac{1}{2} & \frac{3}{2} & \\ & & & 2 \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\left\{(\vec{\sigma}\cdot\vec{n})\left(\frac{1-\varepsilon}{2}\vec{\rho}\cdot\vec{a} + \vec{\rho}\cdot\vec{a}\right)\right\} = \frac{1+\varepsilon}{2}\left\{(\vec{\sigma}\cdot\vec{n})(\vec{\rho}\cdot\vec{a}) + \vec{\sigma}\cdot\vec{n}\frac{1-\varepsilon}{2}\vec{\rho}\cdot\vec{a}\right\}$$

$$\frac{1-\varepsilon}{2}\vec{\rho}\cdot\vec{a} = -\frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a}$$

$$\frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\frac{1-\varepsilon}{2} =$$

$$\frac{1+\varepsilon}{2}(\vec{\sigma}\cdot\vec{a})(\vec{\rho}\cdot\vec{b})\frac{1+\varepsilon}{2}$$

$$\vec{\sigma}\cdot\frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2}\ \frac{\sigma - \rho}{2}$$

$$\vec{\sigma}\cdot\vec{a}\frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\frac{1-\varepsilon}{2} = \begin{pmatrix} a^+ \\ \frac{a_0}{\sqrt{2}} \\ \frac{a_0}{\sqrt{2}} \\ -a^- \end{pmatrix}$$

$\sigma a\ \varepsilon\sigma\cdot a$

$$\frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\frac{1-\varepsilon}{2}\vec{\rho}\cdot\vec{a}\frac{1+\varepsilon}{2}$$

$\sigma a\frac{1-\varepsilon}{2}\sigma a = \frac{1}{2}a^2 + \sigma a\frac{\varepsilon}{2}\sigma a$

$$\left(\frac{1+\varepsilon}{2}\sigma a\frac{1-\varepsilon}{2}\right)\frac{1-\varepsilon}{2}\sigma a\frac{1+\varepsilon}{2}\Big)$$

$$-\frac{1-\varepsilon}{2}\sigma\cdot a\frac{1+\varepsilon}{2}\, a^+a^-\,\frac{1+\varepsilon}{2}\sigma a\frac{1-\varepsilon}{2}$$

$$\frac{1+\varepsilon}{4}\left(\vec{\sigma}\cdot\vec{a}\frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{b}\right)$$

$$\varepsilon n = \begin{pmatrix} 1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1 \end{pmatrix}\begin{pmatrix} \frac{n^0}{\sqrt{2}} & n^+ & & \\ n^- & -\frac{n^0}{\sqrt{2}} & & \\ & & \frac{n^0}{\sqrt{2}} & n^+ \\ & & n^- & -\frac{n^0}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{n^0}{\sqrt{2}} & n^+ & 0 & 0 \\ 0 & 0 & \frac{n^0}{\sqrt{2}} & n^+ \\ n^- & -\frac{n^0}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & n^- & -\frac{n^0}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 0 & a^+ & -a^+ & 0 \\ 0 & -\frac{a^0}{\sqrt{2}} & \frac{a^0}{\sqrt{2}} & 0 \\ 0 & -\frac{a^0}{\sqrt{2}} & \frac{a^0}{\sqrt{2}} & 0 \\ 0 & -a^- & a^- & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 \\ a^- & +\frac{a_0}{\sqrt{2}} & -\frac{a_0}{\sqrt{2}} & -a^+ \\ -a^- & \frac{a_0}{\sqrt{2}} & \frac{a_0}{\sqrt{2}} & a^+ \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2a^+a^- & -\sqrt{2}a^+a_0 & -\sqrt{2}a^+a_0 & -2a^{+2} \\ -\sqrt{2}a^-a^0 & a_0^2 & +a_0^2 & +\sqrt{2}a^0a^+ \\ -\sqrt{2}a^-a^0 & a_0^2 & a_0^2 & \sqrt{2}a^0a^+ \\ 2a^{-2} & \sqrt{2}a_0a^- & \sqrt{2}a_0a^- & 2a^-a^+ \end{pmatrix}$$

$$n^2 = n_1^2+n_2^2+n_3^2 \quad 2n^+n^- + n_0^2 \qquad = \frac{(n_1+in_2)(n_1-in_2)}{2}$$

we want the comb. $n^-n^+ - n^{o2}$

$$\frac{1+\varepsilon}{2} = \begin{pmatrix} 1 & & & \\ & \frac{1}{2} & \frac{1}{2} & \\ & \frac{1}{2} & \frac{1}{2} & \\ & & & 1 \end{pmatrix}$$

$$-3nn^+ = 3\begin{pmatrix} -n^-n^+ & \frac{1}{\sqrt2} n^0 n^+ & \frac{1}{\sqrt2} n^0 n^+ & n^{+2} \\ \frac{n^0}{\sqrt2} n^- & -\frac{n_0^2}{2} & -\frac{n_0^2}{2} & -\frac{n^0}{\sqrt2} n^+ \\ \frac{n^0}{\sqrt2} n^- & -\frac{n_0^2}{2} & -\frac{n_0^2}{2} & -\frac{n_0}{\sqrt2} n^+ \\ n^{-2} & -\frac{n_0}{\sqrt2} n^- & -\frac{n_0}{\sqrt2} n^- & -n^-n^+ \end{pmatrix}$$

$$= -3\, \vec\sigma\cdot\vec n\, \frac{1-\varepsilon}{2}\, \vec\sigma\cdot\vec n$$

$n^-n^+ + \frac{n_0^2}{2} - 3\frac{n_0^2}{2}$

$n^-n^+ - n_0^2$

$2n^+n^- + n_0^2 - 3n^-n^+$

$= n^-n^+ + n_0^2$

$$\frac{1+\varepsilon}{2} n^2 = \begin{pmatrix} 2n^+n^- + n_0^2 & & & \\ & n^-n^+ + \frac{n_0^2}{2} & n^-n^+ + \frac{n_0^2}{2} & \\ & n^-n^+ + \frac{n_0^2}{2} & n^-n^+ + \frac{n_0^2}{2} & \\ & & & 2n^-n^+ + n_0^2 \end{pmatrix}$$

$$\frac{1+\varepsilon}{2} n^2 - 3\, \vec\sigma\cdot\vec n\, \frac{1-\varepsilon}{2}\, \vec\sigma\cdot\vec n = \frac{1+\varepsilon}{2}\left(\vec\sigma\cdot\vec n\, \vec\sigma\cdot\vec n - \vec\sigma\cdot\vec n\, \frac{3}{2}(1-\varepsilon)\, \vec\sigma\cdot\vec n \right)\frac{1+\varepsilon}{2}$$

$$= \frac{1+\varepsilon}{2}\left(\vec\sigma\cdot\vec n \left(1 - \frac{3}{2}(1-\varepsilon)\right) \vec\sigma\cdot\vec n \right)\frac{1+\varepsilon}{2}$$

$1 - \frac{3}{2} + \frac{3\varepsilon}{2}$

$= -\frac{1}{2} + \frac{3\varepsilon}{2}$

$$\frac{1+\varepsilon}{2}\, \vec\sigma\cdot\vec n\, \frac{3\varepsilon-1}{2}\, \vec\sigma\cdot\vec n\, \frac{1+\varepsilon}{2}$$

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$$\Omega = \begin{pmatrix} A & B \\ C & \bar{A} \end{pmatrix}$$

$$\rho_2 \Omega^T \rho_2 = \begin{pmatrix} \bar{A} & -B \\ -C & A \end{pmatrix}$$

is diagonal $\quad \Omega + \rho_2 \Omega^T \rho_2 = \begin{pmatrix} A+\bar{A} & 0 \\ 0 & A+\bar{A} \end{pmatrix} = \; 2\,\mathrm{Tr}\,\Omega$

$$\tfrac{1}{2}\left(\Omega - \rho_2 \Omega^T \rho_2\right) = \begin{pmatrix} \tfrac{1}{2}(A-\bar{A}) & B \\ C & \tfrac{1}{2}(\bar{A}-A) \end{pmatrix}$$

$$= \begin{pmatrix} \tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & \frac{n^0}{\sqrt{2}} n^+ & & \\ & -\tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & -\frac{n_0^2}{2} & \\ & \frac{n_0^2}{2} & -\tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & \end{pmatrix}$$

$$\Pi = \vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & n^+ & -n^+ & 0 \\ 0 & -\frac{n^0}{\sqrt{2}} & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & -\frac{n^0}{\sqrt{2}} & \frac{n^0}{\sqrt{2}} & 0 \\ 0 & -n^- & n^- & 0 \end{pmatrix}$$

$$-\frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{n}\,\frac{1+\varepsilon}{2} - \vec{\sigma}\cdot\vec{n}\,\frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{n} = -\Pi\Pi^\dagger =$$

$$\frac{1}{2}\,\vec{\sigma}$$

$$A = \frac{1+\varepsilon}{2}(\vec{\sigma}\cdot\vec{n})\,\frac{\varepsilon-1}{2}\,\vec{\sigma}\cdot\vec{n}\,\frac{1+\varepsilon}{2}$$

$$= \begin{pmatrix} -n^- n^+ & +\frac{n_0}{\sqrt{2}} n^+ & +\frac{n_0}{\sqrt{2}} n^+ & +n^{+2} \\ +\frac{n^0}{\sqrt{2}} n^- & -\frac{n_0^2}{2} & -\frac{n_0^2}{2} & -\frac{n^0}{\sqrt{2}} n^+ \\ +\frac{n^0}{\sqrt{2}} n^- & -\frac{n_0^2}{2} & -\frac{n_0^2}{2} & -\frac{n^0}{\sqrt{2}} n^+ \\ +n^{-2} & -\frac{n_0}{\sqrt{2}} n^- & -\frac{n_0}{\sqrt{2}} n^- & -n^- n^+ \end{pmatrix}$$

$\cancel{\tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right)}$

$$n^2\left(\frac{1+\varepsilon}{2}\right)$$

$$A' = \frac{1+\varepsilon}{2}(\vec{\sigma}\cdot\vec{n})\,\varepsilon\,(\vec{\sigma}\cdot\vec{n})\,\frac{1+\varepsilon}{2} = \begin{pmatrix} \frac{n_0^2}{2} & \frac{n_0}{\sqrt{2}} n^+ & \frac{n_0}{\sqrt{2}} n^+ & n^{+2} \\ \frac{n_0}{\sqrt{2}} n^- & \frac{1}{2} n^- n^+ & \frac{1}{2} n^- n^+ & -\frac{n^0}{\sqrt{2}} n^+ \\ \frac{n_0}{\sqrt{2}} n^- & \frac{1}{2} n^- n^+ & \frac{1}{2} n^- n^+ & -\frac{n^0}{\sqrt{2}} n^+ \\ n^{-2} & -\frac{n_0}{\sqrt{2}} n^- & -\frac{n_0}{\sqrt{2}} n^- & \frac{n_0^2}{2} \end{pmatrix}$$

$$\frac{A+A'}{2} = \begin{pmatrix} \tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & & & \\ & -\tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & -\tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & \\ & -\tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & -\tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) & \\ & & & \tfrac{1}{2}\left(\frac{n_0^2}{2} - n^- n^+\right) \end{pmatrix}$$

$$n^- n^+ = \frac{(n_1 - i n_2)(n_1 + i n_2)}{2} = \frac{n_1^2 + n_2^2}{2}$$

$$\varepsilon = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \qquad \frac{3\varepsilon}{2} = \begin{pmatrix} \frac{3}{2} & & & \\ & & \frac{3}{2} & \\ & \frac{3}{2} & & \frac{3}{2} \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\, \vec{\sigma}\cdot\vec{n}$$

$$\omega = \frac{3\varepsilon - 1}{2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 1 - 3 \times \frac{1-\varepsilon}{2}$$

$$\vec{\sigma}\cdot\vec{n}$$

$$(a + b\varepsilon) - 2a + b - b + b\varepsilon = (a+b) - b(1-\varepsilon)$$

$$\varepsilon^2 = 1.$$

$$(a + b\varepsilon)^2 = a^2 + b^2 + 2ab\,\varepsilon = \frac{1-3\varepsilon}{2}$$

$$\frac{1}{\sqrt{2}}, \; -\frac{1}{\sqrt{2}}$$

$$2a, \quad a^2 + b^2 = \frac{1}{2}$$

$$2ab = -\frac{3}{2} \qquad ab = -\frac{3}{4}$$

$$\frac{1+\varepsilon}{2}\,\hat{\sigma}\cdot\hat{n} = \frac{1+\varepsilon}{2} \begin{pmatrix} \frac{n^0}{\sqrt{2}} & n^+ & & \\ n^- & -\frac{n^0}{\sqrt{2}} & & \\ & & \frac{n^0}{\sqrt{2}} & n^+ \\ & & n^- & -\frac{n^0}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{n^0}{\sqrt{2}} & n^+ & 0 & 0 \\ \frac{n^-}{2} & -\frac{n^0}{2\sqrt{2}} & \frac{n^0}{2\sqrt{2}} & \frac{n^+}{2} \\ \frac{n^-}{2} & -\frac{n^0}{2\sqrt{2}} & \frac{n^0}{2\sqrt{2}} & \frac{n^+}{2} \\ 0 & 0 & n^- & -\frac{n^0}{\sqrt{2}} \end{pmatrix}$$

$$-\frac{3}{4}$$

$$\omega\,\vec{\sigma}\,\vec{n}\,\frac{1+\varepsilon}{2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{n^0}{\sqrt{2}} & \frac{n^+}{2} & \frac{n^+}{2} & 0 \\ n^- & -\frac{n_0}{2\sqrt{2}} & -\frac{n^0}{2\sqrt{2}} & 0 \\ 0 & +\frac{n_0}{2\sqrt{2}} & \frac{n^0}{2\sqrt{2}} & n^+ \\ 0 & \frac{n^-}{2} & \frac{n^-}{2} & -\frac{n^0}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{n^0}{\sqrt{2}} & \frac{n^+}{2} & \frac{n^+}{2} & 0 \\ -\frac{n^-}{2} & \frac{n^0}{4\sqrt{2}} + \frac{3n^0}{4\sqrt{2}} & +\frac{n^0}{4\sqrt{2}} + \frac{3n^0}{4\sqrt{2}} & \frac{3n^+}{2} \\ \frac{3n^-}{2} & -\frac{3n_0}{4\sqrt{2}} - \frac{n^0}{4\sqrt{2}} & -\frac{3n^0}{4\sqrt{2}} - \frac{n^0}{4\sqrt{2}} & -\frac{n^+}{2} \\ 0 & \frac{n^-}{2} & \frac{n^-}{2} & -\frac{n^0}{\sqrt{2}} \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\,\hat{\sigma}\cdot\hat{n}\;\omega\;\hat{\sigma}\cdot\hat{n}\,\frac{1+\varepsilon}{2} = \begin{pmatrix} \frac{n^0}{\sqrt{2}} & n^+ & 0 & 0 \\ \frac{n^-}{2} & -\frac{n_0}{2\sqrt{2}} & \frac{n_0}{2\sqrt{2}} & \frac{n^+}{2} \\ \frac{n^-}{2} & -\frac{n_0}{2\sqrt{2}} & \frac{n_0}{2\sqrt{2}} & \frac{n^+}{2} \\ 0 & 0 & n^- & -\frac{n^0}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{n^0}{\sqrt{2}} & \frac{n^+}{2} & \frac{n^+}{2} & 0 \\ -\frac{n^-}{2} & +\frac{n^0}{\sqrt{2}} & \frac{n^0}{\sqrt{2}} & \frac{3n^+}{2} \\ \frac{3n^-}{2} & -\frac{n^0}{\sqrt{2}} & -\frac{n^0}{\sqrt{2}} & -\frac{n^+}{2} \\ 0 & \frac{n^-}{2} & \frac{n^-}{2} & -\frac{n^0}{\sqrt{2}} \end{pmatrix}$$

in [illegible]

$$= \begin{pmatrix} \frac{n_0^2 - n^+n^-}{2} & \frac{3n^0}{2\sqrt{2}}n^+ & \frac{3n^0}{2\sqrt{2}}n^+ & \frac{3}{2}{n^+}^2 \\ & \frac{n^-n^+ - n_0^2}{2} & \frac{n^-n^+ - n_0^2}{2} & -\frac{3}{2}\frac{n^0}{\sqrt{2}}n^+ \\ & & \frac{n^-n^+ - n_0^2}{2} & -\frac{3}{2}\frac{n^0}{\sqrt{2}}n^+ \\ & & & \frac{n_0^2 - n^-n^+}{2} \end{pmatrix} \quad T=1$$

$$\frac{n^0}{\sqrt{2}}\frac{n^-}{2} + \frac{n_0 n^-}{4\sqrt{2}} + \frac{3n_0 n^-}{4\sqrt{2}}$$

$$\frac{n^0}{\sqrt{2}}\frac{n^-}{2} + \frac{n_0 n^-}{\sqrt{2}}$$

$$\frac{n^0}{\sqrt{2}}\frac{n^+}{2} + \frac{n^+n^0}{\sqrt{2}}$$

$$\frac{1}{2} + 1 = \frac{3}{2}$$

$$\frac{n^-n^+}{4} - \frac{n_0^2}{4} - \frac{n_0^2}{4} + \frac{n^-n^+}{4} = \frac{n^-n^+ - n_0^2}{2}$$

$$-\frac{3}{4\sqrt{2}}n_0 n^+ - \frac{n_0 n^+}{4\sqrt{2}} - \frac{n_0 n^+}{2\sqrt{2}} = -\left(\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\right) n^0 n^+$$

one must add $\frac{1}{6}(a^-b^+ - a^+b^-)\begin{pmatrix} 1 & \mu & 0 \\ 1 & 1 & \\ & & 0 \end{pmatrix}$

$$\frac{2}{3}\omega = \frac{2}{3} - (1-\varepsilon) = \varepsilon - \frac{1}{3}$$

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{n}\left(\frac{2}{3}\omega\right)\vec{\sigma}\cdot\vec{n}\,\frac{1+\varepsilon}{2} = \begin{pmatrix} \frac{n_0^2 - n^- n^+}{3} & \frac{n^0}{\sqrt{2}}n^+ & \frac{n^0}{\sqrt{2}}n^+ & n^{+2} \\ & \frac{n^- n^+ - n_0^2}{3} & \frac{n^- n^+ - n_0^2}{3} & -\frac{n^0}{\sqrt{2}}n^+ \\ & & \frac{n^- n^+ - n_0^2}{3} & -\frac{n^0}{\sqrt{2}}n^+ \\ & & -\frac{n^0}{\sqrt{2}}n^- & \frac{n_0^2 - n^- n^+}{3} \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\left(\varepsilon-\frac{1}{3}\right)\vec{\sigma}\cdot\vec{b}\,\frac{1+\varepsilon}{2}$$

$$\vec{\sigma}\cdot\vec{a}\,\varepsilon\,\vec{\sigma}\cdot\vec{b} = \vec{\sigma}\cdot\vec{a}\,\vec{\xi}\cdot\vec{b}\,\varepsilon$$
$$= \vec{\xi}\cdot\vec{b}\,\varepsilon\,\vec{\xi}\cdot\vec{a}$$

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\left(\varepsilon-\frac{1}{3}\right)\vec{\sigma}\cdot\vec{b}\,\frac{1+\varepsilon}{2}\,\frac{1+\rho_3}{2} \text{ has the form } \begin{pmatrix} 0 & 0 & b & a \\ 0 & 0 & c & -b \\ 0 & 0 & c & -b \\ 0 & 0 & d & -c \end{pmatrix} \text{ of } T = \frac{3}{2}.$$

$$= \begin{pmatrix} \frac{a_0}{\sqrt{2}} & a^+ & 0 & 0 \\ \frac{a^-}{2} & -\frac{a_0}{2\sqrt{2}} & \frac{a_0}{2\sqrt{2}} & \frac{a^+}{2} \\ \frac{a^-}{2} & -\frac{a_0}{2\sqrt{2}} & \frac{a_0}{2\sqrt{2}} & \frac{a^+}{2} \\ 0 & 0 & a^- & -\frac{a_0}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{\sqrt{2}}{3}b^0 & \frac{1}{3}b^+ & \frac{1}{3}b^+ & 0 \\ -\frac{1}{3}b^- & \frac{\sqrt{2}}{3}b^0 & \frac{\sqrt{2}}{3}b^0 & b^+ \\ b^- & -\frac{\sqrt{2}}{3}b^0 & -\frac{\sqrt{2}}{3}b^0 & -\frac{1}{3}b^+ \\ 0 & \frac{1}{3}b^- & \frac{1}{3}b^- & -\frac{\sqrt{2}}{3}b^0 \end{pmatrix}$$

$$\frac{a^-}{2}\frac{b^+}{3} - \frac{a_0 b_0}{2\cdot 3} - \frac{a_0 b_0}{2\cdot 3} + \frac{a^+ b^-}{2\cdot 3}$$

$$-\frac{a_0 b^+}{2\sqrt{2}} - \frac{a_0 b^+}{2\sqrt{2}\cdot 3} - \frac{a^+ b^0}{3\sqrt{2}}$$

$$-\frac{1}{2\sqrt{2}}\left(1+\frac{1}{3}\right)a_0 b^+$$

$$-\frac{2}{3\sqrt{2}}a_0 b^+ - \frac{a^+ b^0}{3\sqrt{2}} \quad \frac{4}{3}\cdot\frac{1}{2\sqrt{2}}$$

$$= \begin{pmatrix} \frac{1}{3}\left(a_0 b_0 - a^+ b^-\right) & \frac{1}{3}\left(\frac{a_0}{\sqrt{2}}b^+ + \sqrt{2}a^+ b^0\right) & \frac{1}{3}\left(\frac{a_0}{\sqrt{2}}b^+ + \sqrt{2}a^+ b^0\right) & a^+ b^+ \\ & & \frac{1}{6}\left(a^- b^+ + a^+ b^- - 2a_0 b_0\right) & -\frac{1}{3}\left(\frac{a^+ b^0}{\sqrt{2}} + \sqrt{2}a_0 b^+\right) \\ & & \frac{1}{6}\left(a^- b^+ + a^+ b^- - 2a_0 b_0\right) & -\frac{1}{3}\left(\frac{a^+ b^0}{\sqrt{2}} + \sqrt{2}a_0 b^+\right) \\ & & -\frac{1}{3}\left(\sqrt{2}a^- b^0 + \frac{a_0}{\sqrt{2}}b^-\right) & +\frac{1}{3}\left(a_0 b_0 - a^- b^+\right) \end{pmatrix}$$

$$\frac{1}{2}a^- b^+ + \frac{1}{2}b^- a^+ + \frac{1}{2}a^+ b^- + \frac{1}{2}b^+ a^- - a_0 b_0 - a_0 b_0 = a^- b^+ + b^- a^+ - 2a_0 b_0$$

$$a_0 b_0 + b_0 a_0 - a^- b^+ - a^+ b^- = 2a_0 b_0 - a^- b^+ - a^+ b^-$$

$$\frac{a_0}{\sqrt{2}}b^+ + \sqrt{2}a^+ b^0 + \frac{b^0}{\sqrt{2}}a^+ + \sqrt{2}b^+ a^0 = \left(\frac{1}{\sqrt{2}}+\sqrt{2}\right)\left(a^0 b^+ + a^+ b^0\right)$$

$$\frac{a^+ b^0}{\sqrt{2}} + \frac{b^+ a^0}{\sqrt{2}} + \sqrt{2}a_0 b^+ + \sqrt{2}b_0 a^+ = \left(\frac{1}{\sqrt{2}}+\sqrt{2}\right)\left(a^+ b^0 + b^+ a_0\right)$$

$$-\frac{1}{3}\left(\sqrt{2}a^- b^0 + \sqrt{2}b^- a^0 + \frac{a_0}{\sqrt{2}}b^- + \frac{b_0}{\sqrt{2}}a^-\right)$$

$$-\frac{1}{3}\frac{3}{\sqrt{2}}$$

$T = 2$ stat a, b $\qquad \frac{4}{2}\left(\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right) = \frac{3}{\sqrt{2}}$

$$\frac{1+\varepsilon}{2}\ \vec\sigma\cdot\vec a\ (\varepsilon - \tfrac{1}{3})\vec\sigma\cdot\vec b\ \frac{1+\varepsilon}{2} +$$

$$\frac{1+\varepsilon}{2}\left[\vec\sigma\cdot\vec a\tfrac{1}{2}(\varepsilon-\tfrac{1}{3})\vec\sigma\cdot\vec b + \vec\sigma\cdot\vec b\tfrac{1}{2}(\varepsilon-\tfrac{1}{3})\vec\sigma\cdot\vec a\right]\frac{1+\varepsilon}{2}$$

$$= \begin{pmatrix} \frac{1}{6}(2a_0b_0 - a^+b^- - a^-b^+) & \frac{1}{2\sqrt2}(a^0b^+ + a^+b^0) & \frac{1}{\sqrt2}(a^0b^+ + a^+b^0) & a^+b^+ \\ & \frac{1}{6}(a^-b^+ + a^+b^- - 2a_0b_0) & \frac{1}{6}(a^-b^+ + a^+b^- - 2a_0b_0) & -\frac{1}{2\sqrt2}(a^0b^+ + a^+b^0) \\ & & \frac{1}{6}(a^-b^+ + a^+b^- - 2a_0b_0) & -\frac{1}{2\sqrt2}(a^0b^+ + a^+b^0) \\ & & -\frac{1}{\sqrt2}(a^-b^0 + b^-a^0) & \frac{1}{6}(2a_0b_0 - a^+b^- - a^-b^+) \end{pmatrix}$$

$3\varepsilon - 1 \qquad \sqrt2 - 2\sqrt2$

$$\varepsilon - \frac{1}{3} = \frac{3\varepsilon-1}{3} = \frac{2\sqrt2}{3}\,\frac{3\varepsilon-1}{2\sqrt2} \qquad \frac{9}{8} - \frac{1}{8} = 1$$

$$\frac{3}{2\sqrt2}\varepsilon - \frac{1}{2\sqrt2} = a\left(\frac{1}{2\sqrt2} - \frac{3}{2\sqrt2}\varepsilon\right) = a + b\varepsilon \qquad a^2 - b^2 = 1.$$

$a = \cosh\alpha.$

$\frac{b}{a} =$

$$\varepsilon\left(\frac{3}{2\sqrt2} - \frac{1}{2\sqrt2}\varepsilon\right) \qquad \cosh\alpha = \frac{3}{2\sqrt2},\ \sinh\alpha = \frac{1}{2\sqrt2}$$

$$\tanh\alpha = \frac{\sinh\alpha}{\cosh\alpha} = \frac{1}{3}$$

$2\sqrt2\, e^{\varepsilon\frac{\alpha}{2}} = (1+\frac{1}{\sqrt2}) - (1-\frac{1}{\sqrt2})\varepsilon$

$\sqrt2\, e^{\varepsilon\frac{\alpha}{2}} = \frac{(1+\sqrt2)+(1-\sqrt2)\varepsilon}{\sqrt2}$

$e^{\varepsilon\frac{\alpha}{2}} = \frac{(1+\sqrt2)+(1-\sqrt2)\varepsilon}{4}$

$$\frac{2\sqrt2}{3}\, e^{\varepsilon\frac{\alpha}{2}}\,\varepsilon\, e^{\varepsilon\frac{\alpha}{2}} \qquad e^{\varepsilon\alpha} \qquad \tanh\alpha = \frac{1}{3}$$

$(x+\varepsilon y)^2 = 3 - \varepsilon = x^2 + y^2 + 2xy\varepsilon \qquad x^2 + y^2 = 3 \qquad 2xy = -1 \mid xy = -\frac{1}{2}$

$x = 1 + \frac{1}{\sqrt2} \quad y = -1 + \frac{1}{\sqrt2} = -(1 - \frac{1}{\sqrt2})$

$$T = \frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\, e^{\varepsilon\frac{\alpha}{2}}\,\varepsilon\, e^{\varepsilon\frac{\alpha}{2}}\,\vec\sigma\cdot\vec a\,\frac{1+\varepsilon}{2} = U\varepsilon U^\dagger$$

$$\text{if}\quad U = \frac{1+\varepsilon}{2}\vec\sigma\cdot\vec a\, e^{\varepsilon\frac{\alpha}{2}}$$

$$U = \frac{1+\varepsilon}{2}\, \cancel{e^{\frac{\alpha}{2}}}\,\vec\sigma\cdot\vec a\, e^{\varepsilon\frac{\alpha}{2}} = \frac{1+\varepsilon}{2}\,\cancel{e^{\frac{\alpha}{2}}}\,\vec\sigma\cdot\vec a\, e^{\varepsilon\frac{\alpha}{2}} \qquad V = \frac{1+\varepsilon}{2}\vec\sigma\cdot\vec b\, e^{\varepsilon\frac{\alpha}{2}}$$

$$\frac{1+\varepsilon}{2}\vec\sigma = \frac{\vec\sigma}{2} + \frac{\vec\rho}{2}\varepsilon \qquad \underline{T = U\varepsilon V + V\varepsilon U}.$$

Under rotation $\quad U \to e^{i(\rho+\sigma)\cdot\frac{\omega}{2}}U,\quad V \to e^{i(\rho+\sigma)\cdot\frac{\omega}{2}}V$

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$$\begin{matrix} \frac{x}{2} & 0 \\ 0 & \frac{x}{2} \\ 0 & \frac{x}{2} \\ \frac{z}{2} & -y \end{matrix} \qquad \begin{matrix} -x & 0 \\ y & 0 \\ y & 0 \\ z & 0 \end{matrix} \qquad \begin{matrix} x & 0 \\ 0 & x \\ 0 & x \\ z & -2y \end{matrix}$$

$$\begin{pmatrix} -\frac{x}{2} & 0 \\ y & \frac{x}{2} \\ y & \frac{x}{2} \\ \frac{3z}{2} & -y \end{pmatrix}$$

$$\frac{2-2\varepsilon}{4} + \frac{1+\varepsilon}{4} = \frac{3-\varepsilon}{4}$$

$$U \varepsilon U^{\dagger}$$

$$3 - \frac{1+g\cdot\sigma}{2} = \frac{6-1-g\cdot\sigma}{2} = \frac{5-g\cdot\sigma}{2}$$

Hence

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,\frac{3-\varepsilon}{4} = \begin{pmatrix} & & -\frac{x}{2} & \\ & & y & \frac{x}{2} \\ & & y & \frac{x}{2} \\ & & \frac{3z}{2} & -y \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,\frac{3-\varepsilon}{4} = \cancel{\text{[illegible]}}\ \vec{\sigma}\cdot\vec{a}\,\frac{1+\varepsilon}{2} + \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{b}\,\frac{3-\varepsilon}{4}$$

is a combination of

$\vec{\sigma}$, $\vec{g}$, $\vec{g}\times\vec{\sigma}$

$$\frac{1}{3} = \frac{\sinh\alpha}{\cosh\alpha} = \frac{e^{\alpha}-e^{-\alpha}}{e^{\alpha}+e^{-\alpha}} = \frac{e^{2\alpha}-1}{e^{2\alpha}+1} \qquad \text{Let } e^{2\alpha} = x$$

$$\frac{1}{3} = \frac{x-1}{x+1} \qquad x+1 = 3x-3 \qquad 2x = 4 \qquad x = 2.$$

$$e^{2\alpha} = 2 \qquad 2\alpha = \log 2. \qquad \alpha = \tfrac{1}{2}\log 2 = \log\sqrt{2}.$$

$$U = \frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\; e^{\varepsilon\frac{\alpha}{2}} \qquad \cosh\tfrac{\alpha}{2} + \varepsilon\sinh\tfrac{\alpha}{2}$$

$\tanh\alpha = \frac{1}{3}$

$$\cosh\tfrac{\alpha}{2}\left(\frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\,(1+\varepsilon\tanh\tfrac{\alpha}{2}\right)$$

$$e^{2\alpha} = 2 \qquad e^{\alpha} = \sqrt{2}.$$

$$\tanh\frac{\alpha}{2} = \frac{e^{\frac{\alpha}{2}}-e^{-\frac{\alpha}{2}}}{e^{\frac{\alpha}{2}}+e^{-\frac{\alpha}{2}}} = \frac{e^{\alpha}-1}{e^{\alpha}+1}$$

$$\frac{\sqrt{2}(1+\varepsilon)+(1-\varepsilon)}{1+\sqrt{2}}$$

$$\tanh\frac{\alpha}{2} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \qquad 1+\varepsilon\tanh\tfrac{\alpha}{2} = 1+\varepsilon\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}+1+\varepsilon(\sqrt{2}-1)}{\sqrt{2}+1}$$

$$\left(\frac{1-\varepsilon}{2}\right) + \frac{1+\varepsilon}{\sqrt{2}}$$

wrong

$$\boxed{U = \frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\,\frac{1-\varepsilon}{2} + \frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\,\frac{1+\varepsilon}{\sqrt{2}}}$$

$$\frac{n_1+in_2}{\sqrt{2}} = n^+.$$

$$\vec\sigma\cdot\vec n = \sigma_3 n_3 + \sigma_1 n_1 + \sigma_2 n_2.$$

$$\vec\sigma\cdot\vec a = \begin{pmatrix} a_0 & \sqrt{2}a^+ & & \\ \sqrt{2}a^- & -a_0 & & \\ & & a_0 & \sqrt{2}a^+ \\ & & \sqrt{2}a^- & -a_0 \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\,\frac{1-\varepsilon}{2} = \begin{pmatrix} 0 & \frac{a^+}{\sqrt{2}} & -\frac{a^+}{\sqrt{2}} & 0 \\ 0 & -a^0/2 & a^0/2 & 0 \\ 0 & -a^0/2 & a^0/2 & 0 \\ 0 & -\frac{a^-}{\sqrt{2}} & \frac{a^-}{\sqrt{2}} & 0 \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a = \begin{pmatrix} a_0 & \sqrt{2}a^+ & 0 & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a_0}{2} & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a_0}{2} & \frac{a^+}{\sqrt{2}} \\ 0 & 0 & \sqrt{2}a^- & -a_0 \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\,\frac{1+\varepsilon}{2} = \begin{pmatrix} a_0 & \frac{a^+}{\sqrt{2}} & \frac{a^+}{\sqrt{2}} & 0 \\ \frac{a^-}{\sqrt{2}} & 0 & 0 & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & 0 & 0 & \frac{a^+}{\sqrt{2}} \\ 0 & \frac{a^-}{\sqrt{2}} & \frac{a^-}{\sqrt{2}} & -a_0 \end{pmatrix}$$

$$\frac{1+\varepsilon}{2}\,\vec\sigma\cdot\vec a\,\frac{1+\varepsilon}{\sqrt{2}} = \begin{pmatrix} \sqrt{2}a_0 & a^+ & a^+ & 0 \\ a^- & 0 & 0 & a^+ \\ a^- & 0 & 0 & a^+ \\ 0 & a^- & a^- & -\sqrt{2}a_0 \end{pmatrix}$$

$$\frac{1-\varepsilon}{2}\,\vec\sigma\cdot\vec a\,\frac{1+\varepsilon}{2} = \begin{pmatrix} 0 & 0 & & \\ \frac{a^-}{\sqrt{2}} & -\frac{a^0}{2} & -\frac{a^0}{2} & \\ -\frac{a^-}{\sqrt{2}} & \frac{a^0}{2} & \frac{a^0}{2} & \end{pmatrix}$$

$$\left[(1+\sqrt{2})+(1-\sqrt{2})\varepsilon\right]^2 = (1+\sqrt{2})^2+(1-\sqrt{2})^2+2(1+\sqrt{2})(1-\sqrt{2})\varepsilon$$

$$= 1+2+2\sqrt{2}+1+2-2\sqrt{2}+2(1-2)\varepsilon = 6 - 2\varepsilon = 2(3-\varepsilon)$$

$$\omega^2 = \left(\frac{1+\sqrt{2}}{2}+\varepsilon\frac{1-\sqrt{2}}{2}\right)^2 = \frac{2(3-\varepsilon)}{4} = \frac{3-\varepsilon}{2}.$$

$$\omega = \frac{1+\sqrt{2}}{2}+\frac{1-\sqrt{2}}{2}\varepsilon$$

$$\sqrt{2}U = \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\left[\left(1+\frac{1}{\sqrt{2}}\right) - \varepsilon\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

$$U = \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\left(\frac{1+\sqrt{2}}{2}+\varepsilon\frac{1-\sqrt{2}}{2}\right) \qquad T = U\varepsilon U^\dagger$$

$$\varepsilon = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix}$$

$$\omega = \begin{pmatrix} \frac{1+\sqrt{2}}{2}+\frac{1-\sqrt{2}}{2} & & & \\ & \frac{1+\sqrt{2}}{2} & \frac{1-\sqrt{2}}{2} & \\ & \frac{1-\sqrt{2}}{2} & \frac{1+\sqrt{2}}{2} & \\ & & & \frac{1+\sqrt{2}}{2}+\frac{1-\sqrt{2}}{2} \end{pmatrix}$$

$$\frac{2+\sqrt{2}}{2} = 1+\frac{1}{\sqrt{2}}$$

$$-\frac{a_0}{2}\frac{1+\sqrt{2}}{2}+\frac{a_0}{2}\frac{1-\sqrt{2}}{2} = -\frac{a_0}{2}\frac{\sqrt{2}}{2}-\frac{a_0}{2}\frac{\sqrt{2}}{2} = -a_0\frac{\sqrt{2}}{2} = -\frac{a_0}{\sqrt{2}}$$

$$-\frac{a_0}{2}\frac{1-\sqrt{2}}{2}+\frac{a_0}{2}\frac{1+\sqrt{2}}{2} = \frac{a_0\sqrt{2}}{2} = \frac{a_0}{\sqrt{2}}$$

$$\omega = \begin{pmatrix} 1 & & & \\ & \frac{1+\sqrt{2}}{2} & \frac{1-\sqrt{2}}{2} & \\ & \frac{1-\sqrt{2}}{2} & \frac{1+\sqrt{2}}{2} & \\ & & & 1 \end{pmatrix}$$

$$\frac{1+\sqrt{2}}{2}\sqrt{2} = 1+\frac{1}{\sqrt{2}}$$

$$\frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a} = \begin{pmatrix} a_0 & \sqrt{2}a^+ & 0 & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a_0}{2} & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a^0}{2} & \frac{a^+}{\sqrt{2}} \\ 0 & 0 & \sqrt{2}a^- & -a_0 \end{pmatrix}$$

$$U = \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\,\omega = \begin{pmatrix} a_0 & \sqrt{2}a^+ & 0 & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a_0}{2} & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a^0}{2} & \frac{a^+}{\sqrt{2}} \\ 0 & 0 & \sqrt{2}a^- & -a_0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+\sqrt{2}}{2} & \frac{1-\sqrt{2}}{2} & 0 \\ 0 & \frac{1-\sqrt{2}}{2} & \frac{1+\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a_0 & \frac{1+\sqrt{2}}{2}\sqrt{2}a^+ & \frac{1-\sqrt{2}}{2}\sqrt{2}a^+ & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{\sqrt{2}} & \frac{a_0}{\sqrt{2}} & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{\sqrt{2}} & \frac{a_0}{\sqrt{2}} & \frac{a^+}{\sqrt{2}} \\ 0 & \frac{1-\sqrt{2}}{2}\sqrt{2}a^- & \frac{1+\sqrt{2}}{2}\sqrt{2}a^- & -a_0 \end{pmatrix}$$

$$U\cancel{(\sqrt{2})\varepsilon(\sqrt{2})U^\dagger} = \sqrt{2}\begin{pmatrix} a_0 & (1+\frac{1}{\sqrt{2}})a^+ & -(1-\frac{1}{\sqrt{2}})a^+ & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{\sqrt{2}} & \frac{a_0}{\sqrt{2}} & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{\sqrt{2}} & \frac{a_0}{\sqrt{2}} & \frac{a^+}{\sqrt{2}} \\ 0 & -(1-\frac{1}{\sqrt{2}})a^- & (1+\frac{1}{\sqrt{2}})a^- & -a_0 \end{pmatrix} = \begin{pmatrix} \sqrt{2}a_0 & (1+\sqrt{2})a^+ & (1-\sqrt{2})a^+ & 0 \\ a^- & -a_0 & a_0 & a^+ \\ a^- & -a_0 & a_0 & a^+ \\ 0 & (1-\sqrt{2})a^- & (1+\sqrt{2})a^- & -\sqrt{2}a_0 \end{pmatrix}$$

$$\cancel{2U\varepsilon U^\dagger} = \sqrt{2}U^\dagger = \begin{pmatrix} \sqrt{2}a_0 & a^+ & a^+ & 0 \\ (1+\sqrt{2})a^- & -a_0 & -a_0 & (1-\sqrt{2})a^+ \\ (1-\sqrt{2})a^- & a_0 & a_0 & (1+\sqrt{2})a^+ \\ 0 & a^- & a^- & -\sqrt{2}a_0 \end{pmatrix}$$

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$\rho.\sigma = 2\varepsilon - 1$

$T = 2$

$\varepsilon = \frac{1+\rho.\sigma}{2}$

$3\frac{1+\varepsilon}{2}\vec{\sigma}.\vec{a}\left(\vec{\rho}\,\vec{\varepsilon} - \frac{1}{3}\right)\vec{\sigma}.\vec{a}\frac{1+\varepsilon}{2} = \frac{1+\varepsilon}{2}\,\sigma.a\left(1+3\vec{\rho}\vec{\sigma}\right)\sigma.a\,\frac{1+\varepsilon}{2}$

$N_u\,N_1$

$= \vec{\sigma}.\vec{a}\,(\vec{\rho}.\vec{\sigma})\,\vec{\sigma}.\vec{a} + \vec{\rho}.\vec{a}\,\vec{\sigma}.\vec{a}$

~~$\sigma.a\,\varepsilon\,\sigma.a$~~

$= \left(\vec{\sigma}.\vec{a}\;\rho.\sigma + \rho.a\right)\sigma.a \qquad = \begin{pmatrix} \vec{a}\sigma_3 + a_3 & (\sqrt{2}\,\vec{a}\sigma_3^{+} + a^{+}) \\ (\sqrt{2}\sigma^{-}\vec{a} + a^{-} & -\vec{a}\sigma_3 - a_3 \end{pmatrix}\vec{a}$

$N_u N_1 \quad N_1 N_u$

$N_1 N_1 \quad N_u\,N_u$

$= \left[\sigma.\vec{a}\,(2\varepsilon - 1) + \rho.a\right]\sigma.a$

$= 2\sigma.a\,\varepsilon\,\sigma.a - (\sigma.a)^2 + \rho.a\,\sigma.a$

$\sigma.a.\varepsilon\,\sigma.a + \sigma.a\,\rho\,\sigma.a$

$= \varepsilon\,(\rho.a)\,\sigma a + (\sigma.a)(\rho.a)\,\varepsilon$

$\rho.a\,\sigma.a + \varepsilon(\rho.a.\sigma a) + \rho.a\,\sigma.a\,\varepsilon$

$= \frac{1+\varepsilon}{2}(\rho.a)(\sigma.a)\;\sigma.a(1-\varepsilon)\,\sigma.a \quad = \quad \frac{1+\varepsilon}{2}(\rho.a)(\sigma.a) - \sigma.a\frac{1-\varepsilon}{2}\sigma.a + \rho.a\frac{1-\varepsilon}{2}\sigma.a$

$= \frac{1+\varepsilon}{2}(\rho.a)\,\sigma.a + (\rho.a\,\sigma a)\frac{1+\varepsilon}{2} - \sigma.a\frac{1-\varepsilon}{2}\sigma.a - \rho.a\frac{1-\varepsilon}{2}\rho.a$

$= \rho.a\,\sigma.a + \frac{\varepsilon}{2}(\rho.a\,\sigma.a) + (\rho.a\,\sigma.a)\frac{\varepsilon}{2} + \rho.a\frac{1-\varepsilon}{2}\sigma.a - \sigma.a\frac{1-\varepsilon}{2}\rho.a$

$= \frac{\rho.a.\sigma.a}{2} + \frac{\sigma.a\,\rho.a}{2} + \sigma.a\frac{\varepsilon}{2}\sigma.a + \rho.a\frac{\varepsilon}{2}\rho.a - \sigma\frac{a}{2}\sigma.\frac{a}{2} - \rho.\frac{a.\rho a}{2} + \sigma.a\frac{\varepsilon}{2}\sigma.a + \rho.a\frac{\varepsilon}{2}\rho.a$

$= \left\{\rho.a\,\sigma.a - a^2 + \sigma.a\,\varepsilon\,\sigma.a + \rho.a\,\varepsilon\,\rho.a\right\} = \frac{1}{2}\left[\rho - \sigma\right]\vec{a}\,]^2 +$

$\left(\bar{N}_1 N_1\right)\begin{pmatrix}\rho \\ a\end{pmatrix} = \left(\bar{\rho}\,\bar{\sigma}\,\rho + \bar{\sigma}\,\sigma\right)$

$$(\sqrt{2}U)\,\epsilon\,(\sqrt{2}U)^\dagger = \begin{pmatrix} \sqrt{2}a_0 & (1+\sqrt{2})a^+ & (1-\sqrt{2})a^+ & 0 \\ a^+ & -a_0 & a_0 & a^+ \\ a^- & -a_0 & a_0 & a^+ \\ 0 & (1-\sqrt{2})a^- & (1+\sqrt{2})a^- & -\sqrt{2}a_0 \end{pmatrix}\begin{pmatrix} \sqrt{2}a_0 & a^+ & a^+ & 0 \\ (1-\sqrt{2})a^- & a_0 & a_0 & (1+\sqrt{2})a^+ \\ (1+\sqrt{2})a^- & -a_0 & -a_0 & (1-\sqrt{2})a^+ \\ 0 & a^- & a^- & -\sqrt{2}a_0 \end{pmatrix}$$

$(1-2)a^+a^- + (1-2)a^+a^-$
$-2a^+a^-$

$\sqrt{2}\,a_0a^+ + (1+\sqrt{2})a_0a^+ + (1-\sqrt{2})a_0a^+$
$+2\sqrt{2}a_0a^+$

$(1+\sqrt{2})^2a^{+2} + (1-\sqrt{2})^2a^{+2} = (3+2\sqrt{2}+3-2\sqrt{2})a^{+2}$

$2\,a^+ - a_0^2 - a_0^2 + a^+a^-$
$-(1+\sqrt{2})a_0a^+ + (1-\sqrt{2})a_0a^+ - \sqrt{2}a_0a^+$
$-1-\sqrt{2}+1-\sqrt{2}-\sqrt{2} = -3\sqrt{2}$

$a^+a^-(1-2+1-2) + 2a_0^2$

$$= \begin{pmatrix} 2(a_0^2-a^+a^-) & 3\sqrt{2}\,a_0a^+ & 3\sqrt{2}\,a_0a^+ & 6a^{+2} \\ 3\sqrt{2}\,a_0a^- & 2(a^+a^- - a_0^2) & 2(a^+a^- - a_0^2) & -3\sqrt{2}\,a_0a^+ \\ 3\sqrt{2}\,a_0a^- & 2(a^+a^- - a_0^2) & 2(a^+a^- - a_0^2) & -3\sqrt{2}\,a_0a^+ \\ 6a^{-2} & -3\sqrt{2}\,a_0a^- & -3\sqrt{2}\,a_0a^- & 2(a_0^2 - a^+a^-) \end{pmatrix} = \begin{pmatrix} \vec{\sigma}\cdot\vec{a}\;\sigma_3\,\vec{\sigma}\cdot\vec{a} + a_3\,\vec{\sigma}\cdot\vec{a} & \vec{\sigma}\cdot\vec{a}(\sigma_1+i\sigma_2)\vec{\sigma}\cdot\vec{a} + \sigma_1 + \sqrt{2}a^+\vec{\sigma}\cdot\vec{a} \\ a(\sigma_1 - i\sigma_2)a + \sqrt{2}a^-a & -a\sigma_3 a\,a_3 a \end{pmatrix}$$

$\frac{2}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} + \epsilon(1-\frac{1}{\sqrt{2}}) = \sqrt{2} + (1-\frac{1}{\sqrt{2}})(1-\epsilon)$

$$U' = \frac{1+\epsilon}{2}\,\vec{\sigma}\cdot\vec{a}\left[\left(1+\frac{1}{\sqrt{2}}\right) - \epsilon\left(1-\frac{1}{\sqrt{2}}\right)\right] = \frac{1+\epsilon}{2}\,\vec{\sigma}\cdot\vec{a}(1-\epsilon) + \frac{1+\epsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,\frac{1+\epsilon}{\sqrt{2}} = \frac{1+\epsilon}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{a} + \frac{1+\epsilon}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{a}\,\frac{1-\epsilon}{\sqrt{2}}\left(1-\frac{1}{\sqrt{2}}\right)$$

$J = A + B$

$JJ^\dagger = (A+B)(A^\dagger+B^\dagger) = AA^\dagger + BB^\dagger + AB^\dagger + BA^\dagger$

$J^\dagger J = (A^\dagger + B^\dagger)(A+B) = A^\dagger A + B^\dagger B + A^\dagger B + B^\dagger A$

$A \to e^{i(\delta+\sigma)\cdot\frac{\varphi}{2}}A$

$B_0 \to e^{i(\delta+\sigma)\frac{\varphi}{2}}B\,e^{-i\sigma\cdot\frac{\varphi}{2}}$

$= (A^\dagger e^{-i(\delta+\sigma)\cdot\frac{\varphi}{2}} + e^{i\sigma\frac{\varphi}{2}}B^\dagger e^{-i(\delta+\sigma)\frac{\varphi}{2}})(e^{i(\delta+\sigma)\cdot\frac{\varphi}{2}}A + e^{i(\delta+\sigma)\cdot\frac{\varphi}{2}}B\,e^{-i\sigma\cdot\frac{\varphi}{2}})$

$= (A^\dagger e^{\dots}$

$= A^\dagger A + e^{i\sigma\cdot\varphi}B^\dagger B\,e^{-i\sigma\cdot\frac{\varphi}{2}} + A^\dagger B\,e^{-i\sigma\cdot\frac{\varphi}{2}} + e^{i\sigma\cdot\frac{\varphi}{2}}B^\dagger A$

$= A^\dagger A + e^{i\sigma\varphi}B^\dagger B\,e^{-i\sigma\cdot\frac{\varphi}{2}} + Tr\,(A^\dagger B + B^\dagger A)$

$Tr\,J^\dagger J = Tr\,A^\dagger A + Tr\,B^\dagger B + Tr\,(A^\dagger B + B^\dagger A)$

$e^{i(\delta+\sigma)\cdot\frac{\varphi}{2}}B\,e^{-i\sigma\cdot\frac{\varphi}{2}}$

$Tr\,(\Omega(A+B) + (A^\dagger+B^\dagger)\Omega^\dagger$

if $e^{i(\delta+\sigma)\frac{\varphi}{2}}B = B$

$(a^-\;a_0\;a_0\;a^+) + (b^-\;b^-\dots)$

$$\begin{pmatrix} a^+ \\ \frac{a_0}{\sqrt{2}} \\ \frac{a_0}{\sqrt{2}} \\ a^- \end{pmatrix} + \begin{pmatrix} b^+ \\ b_0 \\ b_0 \\ b^- \end{pmatrix}$$

Then $B = \frac{1-\epsilon}{2}B$

$\frac{1-\epsilon}{2}\frac{1+\sigma_3}{2}(u_1 + \sigma_1 u_2)$

$e^{i\sigma\varphi}\cdot\frac{1-\epsilon}{2}\frac{1+\sigma_3}{2}(u_1+\sigma_1 u_2) = \frac{1-\epsilon}{2}\frac{1+\sigma_3}{2}(u_1+\sigma_1 u_2)$

$\vec{\sigma}\cdot\vec{n}\,\frac{1-\epsilon}{2} + \frac{1-\epsilon}{2}\frac{1+\sigma_3}{2}(\bar{\Lambda}p + \sigma_1\bar{\Lambda}n)$

$\frac{1+\epsilon}{2}e^{i(\delta+\sigma)\frac{\varphi}{2}}A\frac{1+\epsilon}{2} + \frac{1+\epsilon}{2}B\,e^{i\sigma\cdot\frac{\varphi}{2}}\Big)\Big(\frac{1-\epsilon}{2}A^\dagger e^{i(\delta+\sigma)\varphi}\frac{1+\epsilon}{2} + e^{\dots}B^\dagger$

$\left(\frac{1-\epsilon}{2}A^\dagger\frac{1+\epsilon}{2} + B^\dagger\frac{1-\epsilon}{2}\right)\left(\frac{1+\epsilon}{2}A\frac{1+\epsilon}{2} + \frac{1+\epsilon}{2}B\frac{1+\epsilon}{2}\right) = \frac{1-\epsilon}{2}A^\dagger A\frac{1+\epsilon}{2} + B^\dagger\frac{1-\epsilon}{2}B$

$$\frac{1}{3}\left(2a_0^2-2\pi^+\pi^-\right)=\frac{1}{\sqrt{2\cdot 3}}\underbrace{\sqrt{\frac{2}{3}}\left(2a_0^2-2\pi^+\pi^-\right)}_{t_0}$$

$$\begin{pmatrix} \frac{1}{3}(2a_0^2-2\pi^+\pi^-) & \sqrt{2}\,\pi_0\pi^+ & \sqrt{2}\,\pi_0\pi^+ & 2\pi^{+2} \\ \sqrt{2}\,\pi_0\pi^- & -\frac{1}{3}(\;\;) & -\frac{1}{3}(\;\;) & -\sqrt{2}\,\pi^0\pi^+ \\ \sqrt{2}\,\pi_0\pi^- & -\frac{1}{3}(\;\;) & -\frac{1}{3}(\;\;) & -\sqrt{2}\,\pi^0\pi^+ \\ 2\pi^{-2} & -\sqrt{2}\,\pi_0\pi^- & -\sqrt{2}\,\pi_0\pi^- & \frac{1}{3}(\;\;) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{6}}t_0 & -t_{+1} & -t_{+1} & 2t_{+2} \\ t_{-1} & -\frac{1}{\sqrt{6}}t_0 & -\frac{1}{\sqrt{6}}t_0 & t_{+1} \\ t_{-1} & -\frac{1}{\sqrt{6}}t_0 & -\frac{1}{\sqrt{6}}t_0 & t_{+1} \\ 2t_{-2} & -t_{-1} & -t_{-1} & \frac{1}{\sqrt{6}}t_0 \end{pmatrix}$$

$$= \rho^+\tau^+ t_{+2} + \frac{1}{\sqrt{2}}(\tau_3\rho^+ + \tau^+\rho_3)t_{+1} + \frac{1}{\sqrt{6}}\left(\rho_3\tau_3 - \frac{\rho^+\tau^- + \rho^-\tau^+}{2}\right)t_0 + \frac{1}{\sqrt{2}}(\tau_3\rho^- + \tau^-\rho_3)t_{-1} + \rho^-\tau^- t_{-2}$$

$$\rho^+ = \frac{1}{\sqrt{2}}(\rho_1 + i\rho_2) = \sqrt{2}\,\frac{\rho_1+i\rho_2}{2} \qquad \rho^+\tau^+ = \begin{pmatrix} 0 & 0 & 0 & 2 \\ & & & 0 \\ & & & 0 \\ & & & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(\tau_3\rho^+ + \tau^+\rho_3) = \tau_3\,\frac{\rho_1+i\rho_2}{2} + \rho_3\,\frac{\tau_1+i\tau_2}{2} = \begin{pmatrix} 0 & \tau_3 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \begin{matrix}0&1\\0&0\end{matrix} & \\ & \begin{matrix}0&-1\\0&0\end{matrix} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ & & & -1 \\ & & & -1 \\ & & & \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(\tau_3\rho^+ + \tau^+\rho_3)(\sqrt{2}\,\pi_0\pi^+) = \begin{pmatrix} & \sqrt{2}\pi_0\pi^+ & \sqrt{2}\pi^0\pi^+ & 2 \\ & & & -\sqrt{2}\pi^0\pi^+ \\ & & & -\sqrt{2}\pi^0\pi^+ \\ & & & \end{pmatrix}$$

$$\rho^+\tau^+\pi^{+2} = \begin{pmatrix} 0 & 0 & 0 & 2\pi^{+2} \\ & & & 0 \\ & & & 0 \\ & & & 0 \end{pmatrix}$$

$$\rho_3\tau_3 - \frac{1}{2}\rho^+\tau^- - \frac{1}{2}\rho^-\tau^+ = \rho_3\tau_3 - \frac{\rho_1+i\rho_2}{2}\,\frac{\tau_1-i\tau_2}{2} - \frac{\rho_1-i\rho_2}{2}\,\frac{\tau_1+i\tau_2}{2}$$

$$\frac{1}{\sqrt{6}}\left(\rho_3\tau_3 - \frac{\rho^+\tau^- + \rho^-\tau^+}{2}\right)T_0 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} - \begin{pmatrix} 0&0 \\ 1&0 \end{pmatrix} - \begin{pmatrix} 0&1 \\ 0&0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & \\ & -1 & -1 & \\ & -1 & -1 & \\ & & & 1 \end{pmatrix}$$

Since, some meson components must ~~always~~ necessarily possess the generalized charge corresponding to the conserved current

$$\frac{1+\varepsilon}{2} \cancel{\Sigma}_\sigma N_\varrho + \Sigma_\sigma \frac{1-\varepsilon}{2} N_\varrho$$

$$\frac{1+\varepsilon}{2}\left(\Sigma_\sigma + \Sigma_\sigma \frac{1-\varepsilon}{2} \right) N_\varrho =$$

$$\frac{1+\varepsilon}{2}\left(\Sigma_\sigma + \Sigma_\sigma \frac{1-\varepsilon}{2} \right) N_\varrho \frac{1-\varepsilon}{2}$$

$$\left(\Sigma_\sigma + \tfrac{1}{2}\Sigma_\sigma - \tfrac{\varepsilon}{2}\Sigma_\varrho \right) \qquad N_\varrho \Sigma_\sigma$$

$$\frac{1+\varepsilon}{2}\left(\tfrac{3}{2}\Sigma_\sigma - \tfrac{1}{2}\Sigma_\varrho \right) N_\varrho \frac{1-\varepsilon}{2}$$

$$\frac{1+\varepsilon}{2}\left(-\tfrac{3}{2} N_\varrho \Sigma_\varrho - \tfrac{1}{2}\Sigma_\varrho N_\varrho \right) \frac{1-\varepsilon}{2}$$

$$-\frac{1+\varepsilon}{2}\left(N_\varrho \Sigma_\varrho + \tfrac{1}{2} N_\varrho \Sigma_\varrho + \tfrac{1}{2}\Sigma_\varrho N_\varrho \right)$$

$$\tfrac{1}{2}(B_0 + \vec{\varrho}\cdot\vec{B})\Sigma_\varrho + \tfrac{1}{2}\Sigma_\varrho(B_0 + \vec{\varrho}\cdot\vec{\varrho})$$

$$\tfrac{1}{2}B_0\Sigma_\varrho + \tfrac{1}{2}(\vec{B}\vec{\Sigma} + \vec{\Sigma}\vec{B})$$

$$\frac{1+\varepsilon}{2}\left(N_\varrho \Sigma_\varrho + \tfrac{1}{2}B_0\vec{\Sigma}_\varrho + \vec{B}\cdot\vec{\Sigma} \right)\frac{1-\varepsilon}{2}$$

$$-\frac{1+\varepsilon}{2}\left(N_\varrho \Sigma_\varrho + \tfrac{1}{2}B_0\Sigma_\varrho \right)\frac{1-\varepsilon}{2}$$

$$(B_0 + B)\Sigma_\varrho = B_0\Sigma_\varrho + \vec{B}_\varrho \times \vec{\Sigma}_\varrho$$

$$\frac{1+\varepsilon}{2}\left(\tfrac{3}{2}B_0 \vec{\varrho}\vec{\Sigma} + \vec{B}_\varrho \times \vec{\Sigma}_\varrho \right)\frac{1-\varepsilon}{2}$$

$$\frac{1+\varepsilon}{2}\left(B_0\vec{\Sigma}_\varrho \pm \vec{B}_\varrho \times \vec{\Sigma}_\varrho \right)\frac{1-\varepsilon}{2}$$

$$\frac{1+\varepsilon}{2}\Sigma_\varrho \vec{N}_\varrho \frac{1-\varepsilon}{2}$$

$$\Sigma_\varrho(B_0 + \vec{\varrho}\cdot\vec{B}) = \Sigma_\varrho B_0 + \Sigma\cdot B + \vec{\Sigma}\times\vec{B}$$

$$\frac{1+\varepsilon}{2}\left(B_0\Sigma + B\times\vec{\Sigma} \right)\frac{1-\varepsilon}{2}$$

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$$\sum_\rho N_\rho = e^{i\vartheta\frac{\omega}{2}} \sum e^{-i\vartheta\frac{\omega}{2}} e^{i\vartheta\frac{\omega}{2}} N_\rho e^{-i\vartheta\frac{\omega}{2}}$$

$$\frac{1-\varepsilon}{2}$$

$\frac{1+\varepsilon}{2} \sum_\rho N_\rho \frac{1-\varepsilon}{2}$

$= \frac{1+\varepsilon}{2} \sum_\rho \bar{N}_\sigma \frac{1-\varepsilon}{2}$

$= \frac{1+\varepsilon}{2} \bar{N}_\sigma \sum_\sigma \frac{1-\varepsilon}{2}$

$\frac{1+\varepsilon}{2} \left(\sum_\rho N_\rho \pm \bar{N}_\sigma \sum_\sigma \right) \frac{1-\varepsilon}{2}$

$$\bar{\Lambda}\ N_\rho \to e^{i\vartheta\frac{\omega}{2}} \Lambda e^{-i\vartheta\frac{\omega}{2}}$$

$$\bar{\Lambda}$$

for $\omega = 2\pi$

$$\frac{1+\varepsilon}{2} e^{i\vartheta\frac{\omega}{2}} \sum_\rho N_\rho e^{-i\vartheta\frac{\omega}{2}} \frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2} e^{i\vartheta\frac{\omega}{2}} e^{i\sigma\frac{\omega}{2}} \sum_\rho N_\rho \frac{1-\varepsilon}{2} = e^{(\rho+\sigma)\frac{\omega}{2}} \frac{1+\varepsilon}{2} \sum_\rho N_\rho \frac{1-\varepsilon}{2}$$

$$\vec{\sum} \left(B_0 + \vec{B} \right) \qquad \frac{1+\varepsilon}{2} \sum_\rho N_\rho$$

$\left(\sum_\rho (a + b\,\varepsilon(\sigma)) F_\rho \right)$

mixture of $T = \frac{3}{2}$ and $T = \frac{1}{2}$ state

$T^{\frac{3}{2}} = 1$

$T^{\frac{1}{2}} = \left(\frac{1}{3} - \delta\sigma\right)$

$$\cancel{1 + \sum 1+\varepsilon} \left(\frac{1+\varepsilon}{2} \sum_\sigma + \sum_\sigma \frac{1-\varepsilon}{2} \right) N_\rho$$

$T = \frac{3}{2}$

$\frac{1+\varepsilon}{2} \left(\sum_\sigma N_\rho + \sum_\sigma \frac{1-\varepsilon}{2} N_\rho \right)$

$T = \frac{1}{2}$

$$\frac{1+\varepsilon}{2} \sum_\rho N_\rho \frac{1-\varepsilon}{2} + \left(\frac{1+\varepsilon}{2} \sum_\sigma + \sum_\sigma \frac{1-\varepsilon}{2} \right) N_\rho$$

$\frac{1+\varepsilon}{2} N_\rho \sum_\sigma \frac{1-\varepsilon}{2}$

$\frac{1+\varepsilon}{2} \sum_\sigma N_\rho \frac{1-\varepsilon}{2}$

$$\frac{1+\varepsilon}{2} \begin{cases} a \sum_\rho N_\rho \frac{1-\varepsilon}{2} + b \left(\sum_\sigma N_\rho + \sum_\sigma \frac{1-\varepsilon}{2} N_\rho \right) \\ a \sum_\rho \frac{3-\varepsilon}{2} N_\rho \frac{1-\varepsilon}{2} + b \sum_\sigma \left(\frac{3-\varepsilon}{2} \right) N_\rho \end{cases}$$

$\gamma_5 = \frac{1 + \vec{\rho}\cdot\vec{\sigma}}{2}$

$\frac{1+\varepsilon}{2} \bar{\sum}_\rho \bar{N}_\rho \frac{1-\varepsilon}{2}$

$T = \frac{1}{2}$

$$\left\{ \sum_\rho \bar{N}_\sigma \frac{1-\varepsilon}{2} \right.$$

$\frac{1+\varepsilon}{2} N_\rho \bar{\sum}_\rho \frac{1-\varepsilon}{2}$

$\sum_\rho$

$\left(\frac{4+\psi^0}{2} + i\frac{4-\psi^0}{2} \right) F_\rho$

$(4+\psi^0) \to (4-\psi^0)$ [illegible]

$\frac{1+\varepsilon}{2} + \frac{1}{2} N_\rho \bar{\sum}_\rho \frac{1-\varepsilon}{2}$

$$\frac{1+\varepsilon}{2} \sum_\sigma N_\rho \frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2} N_\rho \sum_\sigma \frac{1-\varepsilon}{2}$$

$- \frac{1+\varepsilon}{2} N_\rho \bar{\sum}_\rho \frac{1-\varepsilon}{2}$

$\frac{1+\varepsilon}{2} \sum_\rho N_\rho \frac{1-\varepsilon}{2}$

$- \frac{1}{2}\varepsilon N$

$\frac{1+\varepsilon}{2} \left(a \bar{\sum}_\rho \bar{N}_\rho \pm b N_\rho \sum_\rho \right) \frac{1-\varepsilon}{2}$

$$\sum_\sigma N_\rho + \frac{1}{2} \sum_\sigma N_\rho - \frac{1}{2} \sum_\sigma \varepsilon N_\rho$$

$\frac{3}{2} \sum_\sigma N_\rho - \frac{1}{2} \sum_\rho N_\rho$

$\left(\frac{3}{2} \sum_\sigma - \frac{1}{2} \sum_\rho \right) N_\rho$

$$\frac{1+\varepsilon}{2} \frac{3}{2} \sum_\sigma N_\rho \frac{1-\varepsilon}{2} - \frac{1+\varepsilon}{2} \frac{\varepsilon'}{2} \sum_\rho N_\rho \frac{1-\varepsilon}{2}$$

$$\begin{pmatrix} a_0 & \sqrt{2}a^+ & & \\ \sqrt{2}a^- & -a_0 & & \\ & & a_0 & \sqrt{2}a^+ \\ & & \sqrt{2}a^- & -a_0 \end{pmatrix}\begin{pmatrix} l' & m' & l & m \\ m' & -l' & n & -f \\ n' & -l' & n & -l \\ k' & -n' & k & -n \end{pmatrix} = \begin{pmatrix} a_0 l' + \sqrt{2}a^+ n' \\ \sqrt{2}a^- m' \\ \end{pmatrix}$$

$$\begin{matrix} \Xi & \frac{1}{2}p & -\frac{1}{2}p \\ \frac{1}{2}\Xi^- & \frac{1}{2}\Xi^0 & \frac{1}{2}p \\ \frac{1}{2}\Xi^- & \frac{1}{2}\Xi^0 & \end{matrix}$$

Let $\Pi' = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{\pi}\,\frac{1-\varepsilon}{2}$

$S' = \frac{1+\varepsilon}{2}\, S\, \frac{1-\varepsilon}{2}$

$T' = \frac{1+\varepsilon}{2}\, T'\, \frac{1-\varepsilon}{2}$

$\vec{\pi} = \pi\vec{\pi}$

$$S = \bar{\Lambda}\begin{pmatrix} \Xi^0 & & p & \\ & \Xi^0 & & p \\ \Xi^- & & n & \\ & \Xi^- & & n \end{pmatrix}$$

$\Pi_0 K + \vec{\Pi}\vec{K}$

$T = \frac{1}{2}\ (N\,\Xi)$

$e^{i\frac{\sigma\cdot\omega}{2}}\,\Sigma_\beta\, e^{-i\frac{\sigma\cdot\omega}{2}}\, e^{N_s}$

$$J = \begin{pmatrix} 0 & \frac{\pi^+}{\sqrt{2}} & -\frac{\pi^-}{\sqrt{2}} & 0 \\ 0 & -\frac{\pi^0}{2} & \frac{\pi^0}{2} & 0 \\ 0 & -\frac{\pi^0}{2} & \frac{\pi^0}{2} & 0 \\ 0 & -\frac{\pi^-}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} & 0 \end{pmatrix} + \frac{\bar{\Lambda}}{2}\begin{pmatrix} 0 & p & -p & 0 \\ 0 & \frac{\Xi^0 - n}{2} & -\frac{\Xi^0 - n}{2} & 0 \\ 0 & \frac{\Xi^0 - n}{2} & -\frac{\Xi^0 - n}{2} & 0 \\ 0 & -\Xi^- & \Xi^- & 0 \end{pmatrix}^M$$

$$+\frac{1}{2}\begin{pmatrix} 0 & -p\bar{\Sigma}^0 + \frac{\bar{\Sigma}^-}{\sqrt{2}}(n + 3\Xi^0) & -(\quad) & 0 \\ 0 & -(n+\Xi^0)\bar{\Sigma}^0 + \Xi^- \frac{\bar{\Sigma}^-}{\sqrt{2}} + p\frac{\bar{\Sigma}^+}{\sqrt{2}} & -(\quad) & 0 \\ 0 & -(n+\Xi^0)\bar{\Sigma}^0 + \Xi^- \frac{\bar{\Sigma}^-}{\sqrt{2}} + p\frac{\bar{\Sigma}^+}{\sqrt{2}} & -(\quad) & 0 \\ 0 & -(\Xi^0 + 3n)\frac{\bar{\Sigma}^+}{\sqrt{2}} - \Xi^-\bar{\Sigma}^0 & -(\quad) & 0 \end{pmatrix}$$

Swcut:
$$\begin{pmatrix} \bar{\Lambda}p + \Xi^-\Lambda \\ (\Xi^0\Lambda - \bar{\Lambda}n)/2 \\ (\Xi^0\Lambda - \bar{\Lambda}n)/2 \\ \bar{\Lambda}\Xi^- - \Lambda\Xi^- \end{pmatrix}$$

$L = (n^+ + \bar{\Lambda}p + (\bar{p}\,\bar{\Sigma}^0 + \frac{\bar{\Sigma}^-}{\sqrt{2}}(n + 3\Xi^0))\,\omega^+ + c.c.$
$+ ($

$\vec{\Pi}\vec{K}$

$L = \mathrm{Tr}\, J^\dagger J.$

$$e^{\frac{i\vec{\sigma}\cdot\vec{\omega}}{2}}\begin{pmatrix} \Xi^0 & p \\ \Xi^- & n \end{pmatrix} e^{-i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}} = B_0 + \vec{\sigma}\cdot\vec{B}$$

$T = 1$ curnt $J = \vec{\Pi} + \frac{\bar{\Lambda}}{2}\vec{B} + (i\vec{\bar{\Sigma}} \times \vec{B} + b B_0 \vec{\bar{\Sigma}})$ is a complex vector.

out of $T = \frac{3}{2}$ state only

$L = J_1 J_1^\dagger + J_2 J_2^\dagger + J_3 J_3^\dagger$

$$\Pi = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{\pi}\,\frac{1-\varepsilon}{2} \doteq \begin{pmatrix} 0 & \frac{\pi^+}{\sqrt2} & -\frac{\pi^+}{\sqrt2} & 0 \\ 0 & -\frac{\pi^0}{2} & \frac{\pi^0}{2} & 0 \\ 0 & -\frac{\pi^0}{2} & \frac{\pi^0}{2} & 0 \\ 0 & -\frac{\pi^-}{\sqrt2} & \frac{\pi^-}{\sqrt2} & 0 \end{pmatrix}$$

$$S = \begin{pmatrix} & \bar{s}^0 & & s^{++} \\ & & s^0 & s^+ \\ -s^- & & s^0 & \\ & -s^- & & s^0 \end{pmatrix} \qquad T = \begin{pmatrix} t_0' & t'^{+} & t^+ & t^{++} \\ t'^{-} & -t'^{0} & t^- & -t^+ \\ t'^{-} & -t'^{0} & t^- & -t^+ \\ t'^{--} & -t'^{-} & t^0 & -t^- \end{pmatrix}$$

$$= \bar\Lambda \begin{pmatrix} \Xi^0 & & p & \\ & \Xi^0 & & p \\ \Xi^- & & n & \\ & \Xi^- & & n \end{pmatrix}$$

$$J = \Pi + S + T. \qquad L = \mathrm{Tr}\, J^\dagger J.$$

$$\mathrm{Tr}(S^\dagger \Pi + \Pi^\dagger S.)$$

$$\mathrm{Tr}\,\frac{1+\varepsilon}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{\pi^+}{\sqrt2} & -\frac{\pi^0}{2} & -\frac{\pi^0}{2} & -\frac{\pi^+}{\sqrt2} \\ -\frac{\pi^-}{\sqrt2} & \frac{\pi^0}{2} & \frac{\pi^0}{2} & \frac{\pi^-}{\sqrt2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{1+\varepsilon}{2}\bar\Lambda\begin{pmatrix} \Xi^0 & & p & \\ & \Xi^0 & & p \\ \Xi^- & & n & \\ & \Xi^- & & n \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ & -\frac{\pi^0}{2}\bar\Lambda\Xi^0 - \frac{\pi^+}{\sqrt2}\bar\Lambda\Xi^- & & \\ & & -\frac{\pi^-}{\sqrt2}\bar\Lambda p + \frac{\pi^0}{2}\bar\Lambda n & \\ & & & 0 \end{pmatrix}$$

$$= \mathrm{Tr}\left(\frac{1-\varepsilon}{2}\,\vec\sigma\cdot\vec\pi\,\frac{1+\varepsilon}{2}\;\frac{1+\varepsilon}{2}\,F_S\,\frac{1-\varepsilon}{2}\right)$$

$$= \frac{1}{2}\left(\pi^0\bar\Lambda n - \sqrt2\,\pi^-\bar\Lambda p - \sqrt2\,\pi^+\bar\Lambda\Xi^- + \pi^0\bar\Lambda\Xi^0\right)$$

$$\pi^0\bar\Lambda\frac{(n+\Xi^0)}{2} - v$$

$$\begin{pmatrix} \Xi^0 & 0 & p & \\ \frac12\Xi^- & \frac12\Xi^0 & \frac12 n & \cdots \\ \frac12\Xi^- & \frac12\Xi^0 & \frac12 n & \\ 0 & \Xi^- & & \end{pmatrix} \rightarrow e^{i(\varphi-\sigma)\frac{\varepsilon}{2}}\begin{pmatrix} 0 & \frac{p}{2} & -\frac{p}{2} & 0 \\ 0 & \frac{\Xi^0 - n}{4} & -\frac{\Xi^0 - n}{4} & 0 \\ 0 & \frac{\Xi^0 - n}{4} & -\frac{\Xi^0 - n}{4} & 0 \\ 0 & -\frac{\Xi^-}{2} & \frac{\Xi^-}{2} & 0 \end{pmatrix} e^{-i(\varphi-\sigma)\frac{\varepsilon}{2}}$$

$\varepsilon\psi = \psi$

$\mathrm{Tr}\, S_0 T = 0$?

verify.

$$\mathrm{Tr}\left(\frac{1-\varepsilon}{2}\,\Pi\,\frac{1+\varepsilon}{2}\;\frac{1+\varepsilon}{2}\,T\right) = \mathrm{Tr}\left(\frac{1-\varepsilon}{2}\,\frac{1-\varepsilon}{2}\,\Pi\,\frac{1+\varepsilon}{2}\,T\right) = \mathrm{Tr}\left(\frac{1-\varepsilon}{2}\,\Pi\,\frac{1+\varepsilon}{2}\,T\,\frac{1-\varepsilon}{2}\right) \qquad T\varepsilon = T$$

$$S \to \frac{1+\varepsilon}{2}\,S\,\frac{1-\varepsilon}{2}$$

$\mathrm{Tr}\; S^\dagger T = 0$ what about $\mathrm{Tr}\left(\frac{1+\varepsilon}{2}\,S^\dagger\,\frac{1+\varepsilon}{2}\,T\,\frac{1-\varepsilon}{2}\right) = \mathrm{Tr}\left(\frac{1-\varepsilon}{2}\,S^\dagger\,\frac{1+\varepsilon}{2}\,T\right) = \mathrm{Tr}\,\frac{1-\varepsilon}{2}\,S^\dagger T$

$$= \mathrm{Tr}(\varepsilon\, S^\dagger T) = \mathrm{Tr}(S^\dagger T\varepsilon) = \mathrm{Tr}(S^\dagger T).$$

$$\downarrow$$

$$\mathrm{Tr}\left(\frac{1\mp\varepsilon}{2}\,S^\dagger\,\frac{1+\varepsilon}{2}\;\frac{1+\varepsilon}{2}\,T\,\frac{1-\varepsilon}{2}\right) = 0$$

$$T \to e^{i(\rho+\sigma)\frac{\omega}{2}} T e^{-i\sigma\frac{\omega}{2}} e^{-i\rho\frac{\omega}{2}}$$

$$T^\dagger \to e^{i\sigma\frac{\omega}{2}} e^{i\rho\frac{\omega}{2}} T^\dagger e^{-i(\rho+\sigma)\frac{\omega}{2}}$$

$$T^\dagger \Pi \to e^{i\sigma\frac{\omega}{2}} e^{i\rho\frac{\omega}{2}} (T^\dagger_{\frac{1+\varepsilon}{2}} \Pi_{\frac{1-\varepsilon}{2}}$$

$$J = \overbrace{\frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{n}\frac{1-\varepsilon}{2}}^{\Pi} + S_\rho + T$$

$$\Pi \to e^{i(\rho+\sigma)\frac{\omega}{2}} \Pi$$

$$S \to e^{i\rho\frac{\omega}{2}} S e^{-i\rho\frac{\omega}{2}} \quad \text{or} \quad e^{(\rho+\sigma)\frac{\omega}{2}} S e^{-i\rho\frac{\omega}{2}} e^{-i\sigma\frac{\omega}{2}}$$

$(8+4)3$

$$T \to e^{i(\rho+\sigma)\frac{\omega}{2}} T e^{-i\rho\frac{\omega}{2}} e^{-i\sigma\frac{\omega}{2}}$$

$$\cancel{\Pi + S + T} \qquad S_\rho + T \to e^{i(\rho+\sigma)\frac{\omega}{2}} (S_\rho + T) e^{-i\sigma\frac{\omega}{2}} e^{-i\rho\frac{\omega}{2}}$$

M M

$T_{\frac{3}{2}}$ $T_{\frac{3}{2}}$

$6 + 4$

$T^{M_1}_{\frac{1}{2}}$ $T^{M_1}_{\frac{1}{2}}$

$2 + 2$

$+T^1 : 3$

$T^0 : 1$

$$\begin{cases} Tr\,(S_\rho T) = 0 & \text{for arbitrary } S_\rho. \\ T = \varepsilon T \\ T = T\varepsilon \end{cases}$$

$$J = \Pi + (S+T) \to e^{i(\rho+\sigma)\frac{\omega}{2}} \left(\Pi + (S+T) e^{-i\rho\frac{\omega}{2}} e^{-i\sigma\frac{\omega}{2}} \right)$$

$$J^\dagger J \to \Pi^\dagger \Pi + \left(e^{i\rho\frac{\omega}{2}} e^{i\sigma\frac{\omega}{2}} (S+T)^\dagger + \Pi^\dagger \right) \left(\Pi + (S+T) e^{-i\rho\frac{\omega}{2}} e^{-i\sigma\frac{\omega}{2}} \right)$$

$$Tr\,(S+T)^\dagger (S+T) = Tr\, S^\dagger S + Tr\, T^\dagger T$$

$$S = \bar{\Lambda} F_g \rightarrow e^{ig\frac{\omega}{2}} \bar{\Lambda} F e^{-ig\frac{\omega}{2}}$$

$$J = \Pi + S + T$$

$$\Pi \rightarrow e^{i(g+\sigma)\cdot\frac{\omega}{2}} \Pi$$

$$S \rightarrow e^{ig\frac{\omega}{2}} S e^{-ig\frac{\omega}{2}}$$

$$T \rightarrow e^{i(g+\sigma)\frac{\omega}{2}} T e^{-i\sigma\frac{\omega}{2}} e^{-ig\frac{\omega}{2}}$$

$$L = \mathrm{Tr}\, J^{\dagger} J$$

We want $\mathrm{Tr}\; S^{\dagger} T = 0$

$$S = \begin{pmatrix} -s_0 & & s^+ & \\ & -s_0 & & s^+ \\ s^- & & s^0 & \\ & s^- & & s^0 \end{pmatrix}$$

purely scalar matrix

$$S \rightarrow e^{ig\frac{\omega}{2}} S e^{-ig\frac{\omega}{2}}$$

$$S^{\dagger} \rightarrow e^{ig\frac{\omega}{2}} S^{\dagger} e^{-ig\frac{\omega}{2}}$$

$$S = s^+ g^+ + s^0\, 0 \qquad s^0 1 + \vec{g}\cdot\vec{s},$$

purely vectorial matrix

$$S = \begin{pmatrix} -s_0 & s^+ \\ s^- & s_0 \end{pmatrix} \qquad T = \frac{1+\epsilon}{2} \begin{pmatrix} \vec{\sigma}\cdot\vec{a}' & \vec{\sigma}\cdot\vec{a} \\ \vec{\sigma}\cdot\vec{b}' & \vec{\sigma}\cdot\vec{b} \end{pmatrix} \frac{1+\epsilon}{2}.$$

$$\bar{T} \wedge T = 0$$

$$[\sigma^-, (\vec{\sigma}\cdot\vec{b})]_- \cdot [\sigma^+, \vec{\sigma}\cdot\vec{a}]$$

$$\mathrm{Tr}\; S^{\dagger} T = 0.$$

$$S^{\dagger} T = \begin{pmatrix} & \\ & \end{pmatrix}$$

$$\begin{pmatrix} a_3 & a^+ \\ a^- & -a_3 \\ b_3 & b^+ \\ b^- & -b_3 \end{pmatrix}$$

$$b^+ = -a_3$$
$$a^- = b_3$$
$$a = a_3\sigma_3 + b^+$$
$$\sigma^- b = \sigma^- b_3 + \sigma_3 b^+$$
$$\sigma^+ a = \sigma^+ a_3 + \sigma_3 a^-$$
$$b = \sigma_3 b_3 +$$
$$\sigma_2 [\sigma^- b]\sigma_2 = \sigma^+ b_3 \pm \sigma_3 b^+$$
$$[\sigma^+, a] = \sigma^+ a_3 + \sigma_3 a^-$$

$$\left(\frac{1+\epsilon}{2} (\vec{\sigma}\cdot\vec{n}) \frac{1-\epsilon}{2} + S + T \right)$$

$$S^{\dagger} \Lambda \frac{1+\epsilon}{2} \vec{\sigma}\cdot\vec{n} \frac{1-\epsilon}{2}$$

$$e^{ig\frac{\omega}{2}} S_g^{\dagger} e^{-ig\frac{\omega}{2}} e^{i(g+\sigma)\cdot\frac{\omega}{2}} = e^{ig\frac{\omega}{2} + \sigma\frac{\omega}{2}} S_g^{\dagger} \Pi.$$

$$J^\dagger \left(J_1^\dagger e^{-i(\varsigma+\sigma)\cdot\frac{\omega}{2}} + e^{i\sigma\frac{\omega}{2}} J_{\frac{1}{2}}^\dagger e^{-i\varsigma\frac{\omega}{2}} \right) \quad J = \left(e^{i(\varsigma+\sigma)\frac{\omega}{2}} J_{\frac{1+\epsilon}{2}} + e^{i\varsigma\frac{\omega}{2}} J_{\frac{1}{2}} e^{-i\sigma\frac{\omega}{2}} \right.$$

$$= J_1^\dagger J_1 + e^{i\sigma\frac{\omega}{2}} J_{\frac{1}{2}}^\dagger e^{i\sigma\frac{\omega}{2}} J_1$$

$$J J^\dagger = \left(e^{i(\varsigma+\sigma)\frac{\omega}{2}} J_1 \frac{1-\epsilon}{2} J_1^\dagger e^{-i(\varsigma+\sigma)\frac{\omega}{2}} + e^{i(\varsigma+\sigma)\frac{\omega}{2}} J_1 \frac{1-\epsilon}{2} e^{\frac{\omega}{2}} J_{\frac{1}{2}} e^{-i\sigma\frac{\omega}{2}} \right.$$

$$J_1^\dagger e^{-i(\varsigma+\sigma)\cdot\frac{\omega}{2}} \quad e^{i\varsigma\frac{\omega}{2}} e^{i\sigma\frac{\omega}{2}} J_{\frac{1}{2}}$$

$$J \left(\frac{1+\epsilon}{2} \vec{\sigma}\cdot\vec{a} \frac{1-\epsilon}{2} + \frac{1+\epsilon}{2} \vec{\sigma}\cdot\vec{a} \right)$$

$$\frac{1+\epsilon}{2} e^{i\sigma\frac{\omega}{2}} \sigma\cdot a\, e^{-i\sigma\frac{\omega}{2}} \frac{1-\epsilon}{2} +$$

$$\frac{1+\epsilon}{2} \vec{\sigma}\cdot\vec{a} \frac{1-\epsilon}{2} F_\varsigma \to \frac{1+\epsilon}{2} e^{i\sigma\frac{\omega}{2}} \sigma\cdot a\, e^{-i\sigma\frac{\omega}{2}} \frac{1-\epsilon}{2} e^{i\varsigma\frac{\omega}{2}} F_\varsigma e^{-i\varsigma\frac{\omega}{2}}$$

$$= e^{i(\varsigma+\sigma)\frac{\omega}{2}} \frac{1+\epsilon}{2} \sigma\cdot a \frac{1-\epsilon}{2} F_\varsigma e^{-i\sigma\frac{\omega}{2}} e^{-i\varsigma\frac{\omega}{2}}$$

$$e^{i\varsigma\frac{\omega}{2} + i\sigma\frac{\omega}{2}} A e^{-i\sigma\frac{\omega}{2} - i\varsigma\frac{\omega}{2}}$$

$$\frac{1+\epsilon}{2} \vec{\sigma}\cdot\vec{a} \quad F_\varsigma \to \frac{1+\epsilon}{2} e^{i\sigma\frac{\omega}{2}} \vec{\sigma}\cdot\vec{a}\, e^{-i\sigma\frac{\omega}{2}} e^{i\varsigma\frac{\omega}{2}} F_\varsigma e^{-i\varsigma\frac{\omega}{2}}$$

$$A^\dagger A = e^{i\sigma\frac{\omega}{2}\, i\varsigma\frac{\omega}{2}} A^\dagger A e^{-i\sigma\frac{\omega}{2} - i\varsigma\frac{\omega}{2}}$$

$$= \frac{1+\epsilon}{2} e^{i\sigma\cdot\frac{\omega}{2}} e^{i\varsigma\frac{\omega}{2}} \vec{\sigma}\cdot\vec{a}\, F_\varsigma\, e^{-i\sigma\cdot\frac{\omega}{2}} e^{-i\varsigma\frac{\omega}{2}}$$

$\mathrm{Tr}\, A^\dagger A$ is invariant

$$\left(\frac{1+\epsilon}{2} \vec{\sigma}\cdot\vec{a} + \vec{\sigma}\cdot\vec{a} \frac{1-\epsilon}{2} \right) F_\varsigma \to e^{i(\varsigma+\sigma)\cdot\frac{\omega}{2}} \left(\qquad \right) e^{-i\sigma\frac{\omega}{2}} e^{-i\varsigma\frac{\omega}{2}}$$

$\varsigma\omega = u$

$$\vec{\varsigma}\cdot\vec{a}\, F_\varsigma \to e^{i\varsigma\cdot\frac{\omega}{2}} (\vec{\varsigma}\cdot\vec{a}\, F_\varsigma) e^{-i\varsigma\frac{\omega}{2}} \qquad A \frac{1-\epsilon}{2} \to e^{i(\varsigma+\sigma)\frac{\omega}{2}} A e^{i(\varsigma+\sigma)(\omega-u)\ldots}$$

$$\mathrm{Tr}\, \frac{1+\epsilon}{2} A^\dagger A \frac{1+\epsilon}{2}$$

is invariant for $\underline{\omega = u}$.

$$\mathrm{Tr}\, \frac{1-\epsilon}{2} A^\dagger A \frac{1-\epsilon}{2} \text{ is invariant}$$

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$T = \frac{1}{2}$

$$\frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,F_\rho \to \frac{1-\varepsilon}{2}\,e^{i\sigma\frac{\omega}{2}}\,\vec{\sigma}\cdot\vec{a}\,e^{-i\sigma\frac{\omega}{2}}\,\cancel{F_\rho\,e^{i\rho\frac{\omega}{2}}}\,e^{i\rho\frac{\omega}{2}}\,e^{i\rho\frac{\omega}{2}}\,F_\rho$$

$$= \frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,e^{-i\rho\frac{\omega}{2}}\,e^{-i\sigma\frac{\omega}{2}}\,e^{i\rho\frac{\omega}{2}}\,e^{i\rho\frac{\omega}{2}}\,F_\rho$$

$$= \frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\;e^{-i\sigma\frac{\omega}{2}}\,e^{i\rho\frac{\omega}{2}}\,F_\rho$$

$$= \frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\;e^{i\rho\frac{\omega}{2}}\,F_\rho\,e^{-i\sigma\frac{\omega}{2}}$$

$$= \frac{1-\varepsilon}{2}$$

Take $F_\rho \to e$

$$\frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,F_\rho \to \frac{1-\varepsilon}{2}\,e^{i\sigma\frac{\omega}{2}}\,\vec{\sigma}\cdot\vec{a}\,e^{-i\sigma\frac{\omega}{2}}\;e^{i\rho\frac{\omega}{2}}\,F_\rho\,e^{-i\rho\frac{\omega}{2}}$$

$$\to \frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,e^{-i\sigma\frac{\omega}{2}}\,F_\rho\,e^{-i\rho\frac{\omega}{2}}$$

$$\to \frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,F_\rho\,e^{-i\sigma\frac{\omega}{2}-i\rho\frac{\omega}{2}}$$

$-n\ p$

$$\frac{1-\varepsilon}{2}\left(\vec{\sigma}\cdot\vec{a}\,F_\rho\right) \to \frac{1-\varepsilon}{2}\left(\vec{\sigma}\cdot\vec{a}\,F_\rho\right)e^{-i\sigma\frac{\omega}{2}}\,e^{-i\rho\frac{\omega}{2}}$$

$$\left(-\bar{\Lambda}\Xi^-\quad \bar{\Lambda}\Xi^0\quad -\bar{\Lambda}n\quad \bar{\Lambda}p\right)e^{-i\sigma\frac{\omega}{2}}\,e^{-i\rho\frac{\omega}{2}}$$

Let $J_{\frac{1}{2}} \to e^{i\rho\frac{\omega}{2}}\,J_{\frac{1}{2}}\,e^{-i\rho\frac{\omega}{2}}$

$$e^{(\rho+\sigma)i\frac{\omega}{2}}\,\frac{1+\varepsilon}{2}\,J_{1\frac{1}{2}} + \frac{1-\varepsilon}{2}\,J_{\frac{1}{2}}\,e^{-i\sigma\frac{\omega}{2}}\,e^{-i\rho\frac{\omega}{2}}$$

$$J^\dagger_{\frac{1}{2}}\,\frac{1-\varepsilon}{2}\,\gamma_4\,J_1 + J^\dagger_{\frac{1+\varepsilon}{2}}\,\frac{1+\varepsilon}{2}\,J_{\frac{1}{2}} \qquad e^{-i\sigma\frac{\omega}{2}}\,J^\dagger_{\frac{1}{2}}\,e^{i(\rho+\sigma)\frac{\omega}{2}}\,J_1$$

$$\overset{\Lambda}{\overbrace{\Xi^0\ \Xi^-}}$$

$$e^{i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}} e^{i\vec{\rho}\cdot\frac{\vec{u}}{2}} \begin{pmatrix} 0 & p & -p & 0 \\ 0 & n & -n & 0 \\ 0 & \Xi^0 & -\Xi^0 & 0 \\ 0 & \Xi^- & -\Xi^- & 0 \end{pmatrix} \bar{\Lambda}$$

$$T^{\frac{3}{2}} = \frac{1+\varepsilon}{2}\left(\vec{\sigma}\cdot\vec{a} + \vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2}\right)\Gamma_5$$

$$T^{\frac{1}{2}} = \frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,\Gamma_5\, e^{i\cdots}$$

$$\frac{1+\varepsilon}{2} A \frac{1-\varepsilon}{2}$$

$$\vec{u} = \vec{\omega}$$

$$e^{i(\vec{\sigma}+\vec{\rho})\frac{\vec{\omega}}{2}} \begin{pmatrix} 0 & p & -p & 0 \\ 0 & \frac{n+\Xi^0}{2} & -\frac{n+\Xi^0}{2} & 0 \\ 0 & \frac{n+\Xi^0}{2} & -\frac{n+\Xi^0}{2} & 0 \\ 0 & \Xi^- & -\Xi^- & 0 \end{pmatrix} \bar{\Lambda}$$

$$\left(T^{\frac{1}{2}\dagger} + T^{\frac{3}{2}\dagger}\right)\left(T^{\frac{1}{2}} + T^{\frac{3}{2}}\right)$$

$$T^{\frac{1}{2}\dagger} T^{\frac{3}{2}} = 0$$

$$T^{\frac{3}{2}\dagger} T^{\frac{1}{2}} = 0$$

$$J = \left(e^{i(\rho+\sigma)\frac{\omega}{2}} \frac{1+\varepsilon}{2} J_0 \frac{1-\varepsilon}{2} + e^{i(\rho+\sigma)\frac{\omega}{2}} \frac{1+\varepsilon}{2} J_{\frac{3}{2}} e^{-i\sigma\frac{\omega}{2}} e^{-i\rho\frac{u}{2}} + e^{i\sigma\frac{\omega}{2}} e^{i\rho\frac{u}{2}} J_{\frac{1}{2}} \frac{1-\varepsilon}{2} \right)$$

$$J^\dagger J = \left(\frac{1-\varepsilon}{2} J_0^\dagger \frac{1+\varepsilon}{2} e^{-i(\rho+\sigma)\frac{\omega}{2}} + e^{i\sigma\frac{\omega}{2}} e^{i\rho\frac{u}{2}} J^\dagger_{\frac{3}{2}} \frac{1+\varepsilon}{2} e^{-i(\rho+\sigma)\frac{\omega}{2}} + \frac{1-\varepsilon}{2} J^\dagger_{\frac{1}{2}} e^{-i\sigma\frac{\omega}{2}} e^{-i\rho\frac{u}{2}} \right) J$$

$$= \frac{1-\varepsilon}{2} J_0^\dagger J_0 \frac{1-\varepsilon}{2} + e^{+i\rho\frac{u}{2}} e^{i\sigma\frac{\omega}{2}} J^\dagger_{\frac{3}{2}} \frac{1+\varepsilon}{2} J_{\frac{3}{2}} e^{-i\sigma\frac{\omega}{2}} e^{-i\rho\frac{u}{2}} + \frac{1-\varepsilon}{2} J^\dagger_{\frac{1}{2}} J_{\frac{1}{2}} \frac{1-\varepsilon}{2}$$

$$+ \frac{1-\varepsilon}{2} J_0^\dagger \frac{1+\varepsilon}{2} J_{\frac{3}{2}} e^{-i\sigma\frac{\omega}{2}} e^{-i\rho\frac{u}{2}} + e^{i\sigma\frac{\omega}{2}} e^{i\rho\frac{u}{2}} J^\dagger_{\frac{3}{2}} \frac{1+\varepsilon}{2} J_0$$

$$+ \frac{1-\varepsilon}{2} J_0^\dagger \frac{1+\varepsilon}{2} e^{-i(\rho+\sigma)\frac{\omega}{2}} e^{i\sigma\frac{\omega}{2}} e^{i\rho\frac{u}{2}} J_{\frac{1}{2}} \frac{1-\varepsilon}{2} + \ldots$$

$$+ e^{i\sigma\frac{\omega}{2}} e^{i\rho\frac{u}{2}} J^\dagger_{\frac{3}{2}} \frac{1+\varepsilon}{2} e^{i\rho\frac{\omega}{2}} e^{i\rho\frac{u}{2}} J_{\frac{1}{2}} \frac{1-\varepsilon}{2} \qquad \frac{3}{2}\ \frac{1}{2}$$

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$T = \frac{1}{2} \cdot$ matr

$$\Lambda \frac{1-\epsilon}{2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \bar\Lambda & -\bar\Lambda & 0 \\ 0 & -\bar\Lambda & \bar\Lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar\Lambda \frac{1-\epsilon}{2} F_\varsigma = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \bar\Lambda & -\bar\Lambda & 0 \\ 0 & -\bar\Lambda & \bar\Lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Xi^0 & 0 & p & 0 \\ 0 & \Xi^0 & 0 & p \\ \Xi^- & 0 & n & 0 \\ 0 & \Xi^- & 0 & n \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\bar\Lambda \Xi^- & \bar\Lambda \Xi^0 & -\bar\Lambda n & \bar\Lambda p \\ \bar\Lambda \Xi^- & -\bar\Lambda \Xi^0 & \bar\Lambda n & -\bar\Lambda p \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \frac{1-\epsilon}{2} \bar\Lambda F e$$

$e^{i(\varsigma+\sigma)}$

$\bar\Lambda \frac{1-\epsilon}{2} e^{i\varsigma \cdot \frac{\omega}{2}} F_\varsigma$

$= \bar\Lambda \frac{1-\epsilon}{2} e^{-i\sigma\frac{\omega}{2}} F_\varsigma$

$= (\bar\Lambda \frac{1-\epsilon}{2} F_\varsigma) e^{-i\sigma\frac{\omega}{2}} e^{-i\varsigma\frac{\omega}{2}}$

$$J = e^{i(\varsigma+\sigma)\frac{\omega}{2}} J_0 + J_{\frac{1}{2}} + e^{i(\varsigma+\sigma)\frac{\omega}{2}} J_{\frac{3}{2}} \qquad J \to e^{i(\varsigma+\sigma)\frac{\omega}{2}} J$$

$$\operatorname{Tr} J^\dagger J = \operatorname{Tr}\left(J_0^\dagger e^{-i(\varsigma+\sigma)\frac{\omega}{2}} + J^\dagger_{\frac{1}{2}} + J^\dagger_{\frac{3}{2}} e^{-i(\varsigma+\sigma)\frac{\omega}{2}} \right)\left(e^{i(\varsigma+\sigma)\frac{\omega}{2}} J_0 + J_{\frac{1}{2}} + e^{i(\varsigma+\sigma)\frac{\omega}{2}} J_{\frac{3}{2}} \right)$$

$$= J_0^\dagger J_0 + J^\dagger_{\frac{1}{2}} J_{\frac{1}{2}} + J^\dagger_{\frac{3}{2}} J_{\frac{3}{2}} +$$

$$\overset{J_0}{} \qquad \overset{J_{\frac{1}{2}}}{}$$

$$J = \vec\sigma \cdot \vec\Pi \frac{1-\epsilon}{2} + \left(\frac{1+\epsilon}{2} \vec\sigma \cdot \vec\Sigma + \vec\sigma \cdot \vec\Sigma \frac{1-\epsilon}{2} \right) F_\varsigma + \frac{1-\epsilon}{2} \bar\Lambda F_\varsigma$$

$$J = \overset{e^{i(\varsigma+\sigma)}}{\vec\sigma \cdot \vec\Pi \frac{1-\epsilon}{2}} + \left(\overset{e^{i(\varsigma+\sigma)\frac{\omega}{2}}}{\frac{1+\epsilon}{2} \vec\sigma \cdot \vec\Sigma} + \vec\sigma \cdot \vec\Sigma \frac{1-\epsilon}{2} + \frac{1-\epsilon}{2} \bar\Lambda \right) F_\varsigma e^{-i\sigma\frac{\omega}{2}} e^{-i\varsigma\cdot\frac{\omega}{2}}$$

$J \to \qquad e^{i(\varsigma+\sigma)}$

$$F_\varsigma \frac{1-\epsilon}{2} \bar\Lambda \to e^{i\varsigma\frac{\omega}{2}} e^{i\varsigma\frac{\omega}{2}} F_\varsigma \frac{1-\epsilon}{2} \bar\Lambda$$

$$\frac{1+\epsilon}{2} F_\varsigma \frac{1-\epsilon}{2} \bar\Lambda \to \frac{1+\epsilon}{2} F_\varsigma \frac{1-\epsilon}{2} \bar\Lambda$$

$A + B \to e^{i(\varsigma+\sigma)}$

$\frac{1+\epsilon}{2} e^{i\varsigma\cdot\frac{\omega}{2}} e^{i\sigma\frac{\omega}{2}}$

$$F_\varsigma \frac{1-\epsilon}{2} \bar\Lambda = \begin{pmatrix} 0 & -\Xi\Lambda & \Xi\Lambda & 0 \\ 0 & \Xi\Lambda & & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

$A\,T = \frac{1}{2}$ rule.

Consider the subgroup of $\vec{\omega}, \vec{u}$ obtained by $\vec{u} = \vec{\omega}$. Then J is a vector under $\vec{\omega}$.

$$\Lambda \frac{1-\epsilon}{2} = \begin{pmatrix} 0 & & & 0 \\ & \Lambda & -\Lambda & \\ & -\Lambda & \Lambda & \\ 0 & & & 0 \end{pmatrix} \begin{pmatrix} \Xi^0 & & & \\ & \Xi^0 & & \\ \Xi^- & & & \\ & \Xi^- & & \end{pmatrix}$$

Consider $\vec{\sigma}.\vec{a}\frac{1-\epsilon}{2} \cdot \frac{1+a}{2}\left(\frac{3\vec{\sigma}-\vec{\xi}}{2}.\vec{a}\right)$
$= \vec{\sigma}.\vec{a} + \frac{1}{2}(\vec{\xi}\,\vec{\sigma}).\vec{a}$

$$J_s \to e^{i\xi\frac{\omega}{2}} J_s e^{-i\xi\frac{\omega}{2}}$$

$$J_s = \begin{pmatrix} a_0 & \frac{3}{\sqrt{2}}a^+ & -\frac{a^+}{\sqrt{2}} & 0 \\ \frac{a^-}{\sqrt{2}} & -a_0 & a_0 & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -a_0 & a_0 & \frac{a^+}{\sqrt{2}} \\ 0 & -\frac{a^-}{\sqrt{2}} & \frac{3a^-}{\sqrt{2}} & -a_0 \end{pmatrix} \begin{pmatrix} \Xi^0 & 0 & p & 0 \\ 0 & \Xi^0 & 0 & p \\ \Xi^- & 0 & n & 0 \\ 0 & \Xi^- & 0 & n \end{pmatrix}$$

$$= \begin{pmatrix} \Xi^0 a^0 - \Xi^- \frac{a^+}{\sqrt{2}} & \frac{3}{\sqrt{2}}\Xi^0 a^+ & p a_0 - n\frac{a^+}{\sqrt{2}} & \frac{3}{\sqrt{2}} p a^+ \\ \Xi^0 \frac{a^-}{\sqrt{2}} + \Xi^- a_0 & -\Xi^0 a^0 + \Xi^- \frac{a^+}{\sqrt{2}} & p\frac{a^-}{\sqrt{2}} + n a_0 & -p a_0 + n\frac{a^+}{\sqrt{2}} \\ \Xi^0 \frac{a^-}{\sqrt{2}} + \Xi^- a_0 & -\Xi^0 a^0 + \Xi^- \frac{a^+}{\sqrt{2}} & p\frac{a^-}{\sqrt{2}} + n a_0 & -p a_0 + n\frac{a^+}{\sqrt{2}} \\ \frac{3}{\sqrt{2}}\Xi^- a^- & -\Xi^0\frac{a^-}{\sqrt{2}} - \Xi^- a_0 & \frac{3}{\sqrt{2}} n a^- & -p\frac{a^-}{\sqrt{2}} - n a_0 \end{pmatrix}$$

Transformation property: $J \to e^{i(\xi+\sigma).\frac{\omega}{2}} J e^{-i\vec{\sigma}\frac{\vec{\omega}}{2}} e^{-i\vec{\xi}.\frac{\vec{a}}{2}}$

J': strangeness nonchanging current. $\vec{a}\,\vec{\sigma}\,\frac{1-\epsilon}{2} \to e^{i(\xi+\sigma).\frac{\omega}{2}} J$

Consider $J = (J' + J_s)\frac{1-\epsilon}{2} = \begin{pmatrix} 0 & \frac{a^+}{\sqrt{2}} & -\frac{a^-}{\sqrt{2}} & 0 \\ 0 & -\frac{a_0}{2} & \frac{a_0}{2} & 0 \\ 0 & -\frac{a_0}{2} & \frac{a_0}{2} & 0 \\ 0 & -\frac{a^-}{\sqrt{2}} & \frac{a^-}{\sqrt{2}} & 0 \end{pmatrix} + \begin{pmatrix} 0 & \frac{3}{2\sqrt{2}}\left(\Xi^0 a^+ + p a^0 - n\frac{a^+}{\sqrt{2}}\right) & -(\ ) & 0 \\ 0 & - & & \\ 0 & & & \\ 0 & & & \end{pmatrix}$

$$J' + \begin{pmatrix} 0 & \frac{1}{2}\left(-p a_0 + \frac{a^+}{\sqrt{2}}(n + 3\Xi^0)\right) & -(\ ) & 0 \\ 0 & \frac{1}{2}\left(-a_0(n+\Xi^0) + \Xi^-\frac{a^+}{\sqrt{2}} + p\frac{a^-}{\sqrt{2}}\right) & -(\ ) & 0 \\ 0 & \frac{1}{2}\left(-a_0(n+\Xi^0) + \Xi^-\frac{a^+}{\sqrt{2}} + p\frac{a^-}{\sqrt{2}}\right) & -(\ ) & 0 \\ 0 & \frac{1}{2}\left(-\frac{a^-}{\sqrt{2}}(\Xi^0 + 3n) - \Xi^- a_0\right) & -(\ ) & 0 \end{pmatrix}$$

General form of the $T=\frac{3}{2}$ matrix

$$T^{3/2} = \vec{\sigma}\cdot\vec{a}\,\rho_1 + [\sigma^+, \vec{\sigma}\cdot\vec{a}] + \sigma^- ,$$

$$T^{3/2} = \left[\vec{\sigma}\cdot\vec{a} + \left([\sigma^-, \vec{\sigma}\cdot\vec{a}] + \sigma^+ b\right)\rho_1\right]\frac{1\mp\rho_3}{2}$$

we have $T^{3/2} = \varepsilon T^{3/2}$

$$T^{\frac{3}{2}} \to e^{i(\rho+\sigma)\cdot\frac{\omega}{2}}\, T^{\frac{3}{2}}\, e^{-i\sigma\cdot\frac{\omega}{2}}$$

$$e^{i\sigma\cdot\frac{\omega'}{2}}\, e^{i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}$$

6 complex

$$e^{i(\rho+\sigma)\frac{\omega}{2}}\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ & \end{pmatrix}$$

4×4 matrix

$$\begin{pmatrix} p & \Xi^0 \\ n & \Xi^- \end{pmatrix} e^{i\vec{\sigma}\cdot\vec{a}}$$

$\frac{3}{\sqrt{2}}\,\Xi^0 a^+$

$-\Xi^0 a_0 + \Xi^- \frac{a^+}{\sqrt{2}}$

$-\Xi^- a_0 + \Xi^- \frac{a^+}{\sqrt{2}}$

$-\Xi^0 \frac{a^-}{\sqrt{2}}$

$$e^{i(\rho+\sigma)\cdot\frac{\vec{\omega}}{2}}\begin{pmatrix} -n\frac{a^+}{\sqrt{2}} + p a_0 & \frac{3}{\sqrt{2}}\Xi^0 a^+ \\ \frac{p a^-}{\sqrt{2}} + n a_0 & -\Xi^0 a_0 + \Xi^- \frac{a^+}{\sqrt{2}} \\ p a^- + n a_0 & \vdots \\ \frac{3}{2} n a^- & \end{pmatrix} e^{i\vec{\sigma}\cdot\vec{\omega}}$$

$$e^{i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}\, e^{i\vec{\rho}\cdot\frac{\vec{\omega}}{2}}$$

$$e^{i(\rho+\sigma)\frac{\omega}{2}}\begin{pmatrix} -\Xi^-\frac{a^-}{\sqrt{2}} + \Xi^0 a_0 & \frac{3}{\sqrt{2}}\Xi^0 a^+ & \frac{n a^+}{\sqrt{2}} + p a_0 & \frac{3}{\sqrt{2}} p a^+ \\ & & & \\ & & & \end{pmatrix} e^{i\sigma\frac{\omega}{2}}\, e^{i\rho\frac{\omega}{2}}.$$

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a, b, c complex

$$\hat{\sigma}\cdot\hat{z} = \vec{\sigma}\cdot(\vec{x}+i\vec{y}) = \begin{pmatrix} x_3+iy_3 & (x_1+ix_2)+i(y_1+iy_2) \\ (x_1-ix_2)+i(y_1-iy_2) & -(x_3+iy_3) \end{pmatrix} = \begin{pmatrix} a & b \\ c & -a \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & -a \\ a' & b' \\ c' & -a' \end{pmatrix}$$

$c = a'$ $\quad b' = -a$

$[\sigma^+, \sigma_3] = \sigma^+$
$[\sigma^-, \sigma_3] = \sigma^-$
$[\sigma^+, \sigma^+] = 0$ $\quad [\sigma^+, \sigma^-] = \sigma_3$

$\begin{matrix} a & b \\ c & -a \end{matrix}$

$\begin{matrix} a & b \\ c & -a \\ c & -a \\ a' & -c \end{matrix}$

$\begin{pmatrix} c & -a \\ c' & -c \end{pmatrix}$

$(x_1+ix_2) = -i(y_1+iy_2)$

$x_1 = y_2$

$x_2 = -y_1$

$H, \quad H'$

$$\begin{pmatrix} \vec{a} \\ \vec{b} \end{pmatrix} = \varepsilon \begin{pmatrix} \vec{a} \\ \vec{b} \end{pmatrix}$$

$$\begin{cases} H'_3 = H^- \\ H_3 = H'^+ \end{cases}$$

$\vec{a} = \begin{pmatrix} a & b \\ c & -a \end{pmatrix}$

$[\sigma^+, \vec{a}] = [\sigma^+, a\sigma_3 + b\sigma^+ + c\sigma^-]$
$= a\sigma^+ + c\sigma_3$

$(a\sigma_3 + b\sigma^+ + c\sigma^-)\rho^+ + (a'\sigma_3 -$

$$T^{3/2} = (a\sigma_3 + b\sigma^+ + c\sigma^-)\rho^+ + (c\sigma_3 - a\sigma^+ + c'\sigma^-)\frac{1+\rho_3}{2}.$$

$$= b\sigma^+\rho^+ + a\left(\sigma_3\rho^+ - \sigma^+\frac{1+\rho_3}{2}\right) + c\left(\sigma^-\rho^+ + \sigma_3\frac{1+\rho_3}{2}\right) + c'\sigma^-\frac{1+\rho_3}{2}.$$

$$= \left\{b\sigma^+\rho_1 + a(\sigma_3\rho_1 - \sigma^+) + c(\sigma^-\rho_1 + \sigma_3) + c'\sigma^-\right\}\frac{1+\rho_3}{2}.$$

$$= \left\{(b\sigma^+ + a\sigma_3 + c\sigma^-)\rho_1 + (-a\sigma^+ + c\sigma_3 + c'\sigma^-)\right\}\frac{1+\rho_3}{2}.$$

$= (b\sigma^+ +$

$\sigma^+(b\sigma^+ + a\sigma_3 + c\sigma^-) = a\sigma^+\sigma_3 + c\sigma^+\sigma^-$

$$T^{3/2} = \underline{\left(\vec{a}\rho_1 + [\sigma^+, \vec{a}] + \sigma^- c'\right)\frac{1+\rho_3}{2}} \qquad \frac{1+\varepsilon}{2}\left(\vec{a} + \vec{b}\rho_1\right)\frac{1+\rho_3}{2}.$$

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$$\vec{\sigma}\cdot\vec{a}\,\overbrace{(n+g_1 p)}^{N_g}\frac{1-g_3}{2} = \begin{pmatrix} \vec{\sigma}\cdot a & \\ & \vec{\sigma}\cdot a \end{pmatrix}\begin{pmatrix} p\,0 & p \\ & p \\ p\,0\,n & \\ & n \end{pmatrix} = \begin{pmatrix} 0 & p\,\vec{\sigma}\cdot\vec{a} \\ 0 & n\,\vec{\sigma}\cdot\vec{a} \end{pmatrix}$$

Contains both $T=\frac{1}{2}$ and $T=\frac{3}{2}$ states.

$$\vec{\sigma}\cdot\vec{a} = \frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a} + \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}$$

$\frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\,N_g$ contains only $T=\frac{1}{2}$ states.

$$\vec{\sigma}\cdot\vec{a} = \frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a} + \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}$$

$$\frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\,N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_3}{2} & -\frac{a_3}{2} & -\frac{a^+}{\sqrt{2}} \\ -\frac{a^-}{\sqrt{2}} & \frac{a_3}{2} & \frac{a_3}{2} & \frac{a^+}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & p & p \\ 0 & 0 & & \\ 0 & 0 & n & \\ 0 & 0 & & n \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 & \frac{a^-}{\sqrt{2}}p - \frac{a_3}{2}n \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\vec{\sigma}\cdot\vec{a} = \vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2} + \vec{\sigma}\cdot\vec{a}\,\frac{1+\varepsilon}{2}$$

so that $\vec{\sigma}\cdot\vec{a} = \frac{1-\varepsilon}{2}\left(\vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2} + \vec{\sigma}\cdot\vec{a}\,\frac{1+\varepsilon}{2}\right) + \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}$.

$$\lambda T^{\frac{3}{2}} + \mu T^{\frac{1}{2}} = \lambda\frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a} + \lambda\vec{\sigma}\cdot\vec{a}\frac{1-\varepsilon}{2} + \mu\frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a}$$

$$= \frac{\lambda+\mu+(\lambda-\mu)\varepsilon}{2}\vec{\sigma}\cdot\vec{a} + \lambda\,\vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2}$$

$$= \frac{2\lambda+\mu}{2}\vec{\sigma}\cdot\vec{a} + \frac{\lambda-\mu}{2}\varepsilon\vec{\sigma}\cdot\vec{a} - \frac{\lambda}{2}\vec{\sigma}\cdot\vec{a}\,\varepsilon$$

$$e^{i(g+\sigma)\frac{\omega}{2}}\;e^{-i\sigma\frac{\omega}{2}}\;\frac{1-g_3}{2}$$

$$e^{ig\frac{\omega}{2}}(\quad)$$

$$\lambda\frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a} + \lambda\frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a} + \mu\cdot 1$$

$$\lambda\,\vec{\sigma}\cdot\vec{a} + \frac{\lambda}{2}\left(\vec{\sigma}\cdot\vec{a} - \vec{\sigma}\cdot\vec{a}\,\varepsilon\right) + \mu\frac{1-\varepsilon}{2}\vec{\sigma}\cdot\vec{a}$$

$$e^{i\sigma\frac{\omega}{2}}\vec{\sigma}\cdot\vec{a}\;e^{-i\sigma\frac{\omega}{2}}\;e^{ig\frac{\omega}{2}}N$$

$$e^{ig\frac{\omega}{2}}(\quad)$$

$$a\frac{1+\varepsilon}{2}\ldots$$

$$T^{\frac{3}{2}} + T^{\frac{1}{2}} =$$

$$N \to e^{ig\frac{\omega}{2}}$$

$$(\vec{g}\cdot\vec{a}) \to e^{ig\frac{\omega}{2}}(g\cdot a)\,e^{-ig\frac{\omega}{2}}$$

$$(\vec{g}\cdot\vec{a})N \to e^{ig\frac{\omega}{2}}(g\cdot a)N$$

$$T = \frac{1}{2} \quad ! \quad \vec{\rho}\cdot\vec{a}\,(n+\rho_1 p)\frac{1+\rho_3}{2} = \varepsilon\,\vec{\sigma}\cdot\vec{a}\,\varepsilon\,(n+\rho_1 p)\frac{1+\rho_3}{2}$$

$$\begin{pmatrix} a_0 & 0 & \sqrt{2}a^+ & 0 \\ 0 & a_0 & 0 & \sqrt{2}a^+ \\ \sqrt{2}a^- & 0 & -a_0 & 0 \\ 0 & \sqrt{2}a^- & 0 & -a_0 \end{pmatrix}\begin{pmatrix} 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{pmatrix} = \begin{pmatrix} 0 & 0 & pa_0+\sqrt{2}ma^+ & 0 \\ 0 & 0 & 0 & pa_0+\sqrt{2}ma^+ \\ 0 & 0 & \sqrt{2}pa^- - ma_0 & 0 \\ 0 & 0 & 0 & \sqrt{2}pa^- - ma_0 \end{pmatrix}$$

$$T^{\frac{3}{2}} = \left(\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a} + \vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2}\right)(m+\rho_1 p)\frac{1-\rho_3}{2}$$

$$T_\rho^{\frac{1}{2}} = \left(\varepsilon\,\vec{\sigma}\cdot\vec{a}\,\varepsilon\right)(m+\rho_1 p)\frac{1-\rho_3}{2}$$

$\vec{\rho}$:

$$\lambda T^{\frac{3}{2}} + \mu T^{\frac{1}{2}} = \left(\lambda\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a} + \lambda\,\vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2} + \mu\,\varepsilon\,\vec{\sigma}\cdot\vec{a}\,\varepsilon\right)(m+\rho_1 p)\frac{1-\rho_3}{2}$$

$$T_\sigma^{\frac{1}{2}} = \varepsilon T^{\frac{1}{2}}\varepsilon = (\vec{\sigma}\cdot\vec{a})(n+\sigma_1 p)\left(\frac{1-\sigma_3}{2}\right) = (\vec{\sigma}\cdot\vec{a})\begin{pmatrix} 0 & p \\ 0 & m\end{pmatrix} = \begin{pmatrix} a_0 & \sqrt{2}a^+ \\ \sqrt{2}a^- & -a_0\end{pmatrix}\begin{pmatrix} 0 & p \\ 0 & m\end{pmatrix} = \begin{pmatrix} 0 & pa_0+\sqrt{2}ma^+ \\ 0 & \sqrt{2}a^-p - ma_0\end{pmatrix}$$

$$e^{\sigma\cdot a}\left(e^{\rho\cdot a}\; e^{\sigma\cdot a}\right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & pa_0+\sqrt{2}ma^+ \\ 0 & 0 & 0 & \sqrt{2}pa^- - ma_0 \end{pmatrix}$$

A is eigenstate of $\frac{1+\varepsilon}{2}$.

A = 2×2 matrix.

$$\left(\begin{array}{l} \frac{1+\varepsilon}{2}A = A \\ \quad 2 \text{ conditions} \\ \mathrm{Tr}\,A = 0 \quad 1 \text{ cond.} \\ \mathrm{Tr}\,\rho_1 A = 0 \end{array}\right.$$

then $a = -d$, $c = -g$

a	b
c	d
e	d
f	g

$A + \bar{A} = 0 \qquad A = -\bar{A}$

σ_2

$\bar{A} = \sigma_2 \overset{T}{A} \sigma_2 \qquad (A = \sigma_2 \overset{T}{A} \sigma_2)$

A

$\vec{A}\cdot\vec{\sigma} = \begin{pmatrix} (E_3+iH_3) & \cdots \\ b & -(E_3+iH_3)\end{pmatrix}$

$\bar{A} \quad B \quad \bar{A} \quad \bar{B} \quad \begin{pmatrix}\vec{a} \\ \vec{b}\end{pmatrix}$

$\vec{\sigma}\cdot\vec{a} + \frac{i}{2}\vec{\rho}\times\vec{\sigma}\cdot\vec{a}$

$$\begin{pmatrix} a_0 & \frac{3}{\sqrt{2}}a^+ & -\frac{a^+}{\sqrt{2}} & 0 \\ \frac{a^-}{\sqrt{2}} & -a_0 & a_0 & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -a_0 & a_0 & \frac{a^+}{\sqrt{2}} \\ 0 & -\frac{a^-}{\sqrt{2}} & \frac{3a^-}{\sqrt{2}} & -a_0 \end{pmatrix}\begin{pmatrix} 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\frac{na^+}{\sqrt{2}}+pa_0 & \frac{3}{\sqrt{2}}pa^+ \\ 0 & 0 & \frac{pa^-}{\sqrt{2}}+na_0 & -pa_0+n\frac{a^+}{\sqrt{2}} \\ 0 & 0 & \frac{pa^-}{\sqrt{2}}+na_0 & -pa_0+\frac{na^+}{\sqrt{2}} \\ 0 & 0 & \frac{3}{\sqrt{2}}pa^- & -p\frac{a^-}{\sqrt{2}}-na_0 \end{pmatrix}$$

$\vec{\sigma}\cdot\vec{a} + \frac{i}{2}\vec{\rho}\times\vec{\sigma}\cdot\vec{a}$

$$T^{\frac{3}{2}} = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\left(1+\frac{1-\varepsilon}{2}\right)(n+\rho_1 p)\frac{1-\rho_3}{2} = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,\frac{3-\varepsilon}{2}\,(n+\rho_1 p)\frac{1-\rho_3}{2}.$$

$$T^{\frac{1}{2}} = \vec{\rho}\cdot\cancel{\vec{a}}\,(n+\rho_1 p)\frac{1-\rho_3}{2}.$$

$$T^{\frac{1}{2}} = \left(\frac{1-\varepsilon}{\cancel{2}}\right)\vec{\sigma}\cdot\vec{a}\,(n+\rho_1 p)\frac{1-\rho_3}{2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & pa^- - na_0 & pa_0+\sqrt{2}a^+ \\ 0 & 0 & -pa^-+na_0 & -pa_0+\sqrt{2}na^+ \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(1-\varepsilon)\,\vec{\sigma}\cdot\vec{a} = (1-\varepsilon)\,\vec{\sigma}\cdot\vec{a}\,\frac{1+\varepsilon}{2}.$$

$$\frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & -\frac{a_0}{2} & -\frac{a^+}{\sqrt{2}} \\ -\frac{a^-}{\sqrt{2}} & +\frac{a_0}{2} & +\frac{a_0}{2} & +\frac{a^+}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T^{\frac{3}{2}} = \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\left(1+\frac{1-\varepsilon}{2}\right)(n+\rho_1 p)\frac{1-\rho_3}{2}.$$

$$\vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2} = \begin{pmatrix} 0 & \frac{a^+}{\sqrt{2}} & -\frac{a^+}{\sqrt{2}} & 0 \\ 0 & -\frac{a_0}{2} & \frac{a_0}{2} & 0 \\ 0 & -\frac{a_0}{2} & \frac{a_0}{2} & 0 \\ 0 & -\frac{a^-}{\sqrt{2}} & \frac{a^-}{\sqrt{2}} & 0 \end{pmatrix}$$

$= \frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2}$

$= (\varepsilon-\sigma)\cdot\vec{a}\,\frac{1+\varepsilon}{2}$

$$T^{\frac{1}{2}} = \left(\frac{1-\varepsilon}{2}\right)\vec{\sigma}\cdot\vec{a}\,\cancel{(1+\varepsilon)}\,(n+\rho_1 p)\frac{1-\rho_3}{2}.$$

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a} = \begin{pmatrix} a_0 & \sqrt{2}a^+ & 0 & 0 \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a_0}{2} & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & -\frac{a_0}{2} & \frac{a_0}{2} & \frac{a^+}{\sqrt{2}} \\ 0 & 0 & \sqrt{2}a^- & -a_0 \end{pmatrix}$$

$\vec{\sigma}\cdot\vec{a} + \frac{i}{2}\vec{\rho}\times\vec{\sigma}\cdot\vec{a}$

$\frac{1+\varepsilon}{2} = \varepsilon^+$

$\frac{1-\varepsilon}{2} = \varepsilon^-$

$(\varepsilon^+)^2 = \varepsilon^+ \quad (\varepsilon^-)^2 = \varepsilon^-$

$\varepsilon^+ + \varepsilon^- = 1$

$\varepsilon = \varepsilon^+ - \varepsilon^-$

$\varepsilon^+\varepsilon^- = 0$

$\frac{1+\varepsilon}{2}\,\vec{\sigma}\,\vec{a}\,\frac{1-\varepsilon}{2}$

$\frac{1-\varepsilon}{2}\,\vec{\sigma}\,\vec{a}\,\frac{1+\varepsilon}{2}$

$$T^{\frac{3}{2}} = \left(\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a} + \vec{\sigma}\cdot\vec{a}\,\frac{1-\varepsilon}{2}\right)(n+\rho_1 p)\frac{1-\rho_3}{2}$$

$$T^{\frac{1}{2}} = \left(\frac{1-\varepsilon}{2}\,\vec{\sigma}\cdot\vec{a}\,\frac{1+\varepsilon}{2}\right)(n+\rho_1 p)\frac{1-\rho_3}{2}.$$

$\frac{1+\varepsilon}{2} = \varepsilon^+, \quad \frac{1-\varepsilon}{2} = \varepsilon^-$

$\frac{1-\varepsilon}{2}\,\vec{\sigma}\,\vec{a}\,\frac{1-\varepsilon}{2} = 0 \qquad \varepsilon^-\vec{\sigma}\,\vec{a}\,\varepsilon^- = 0$

$\vec{\sigma}\,\vec{a}\,\varepsilon^- = \varepsilon^+\vec{\sigma}\,\vec{a}\,\varepsilon^-$

$$\lambda_1\vec{\sigma}\cdot\vec{a} + \lambda_2\vec{\rho}\cdot\vec{a} = \lambda_1\vec{\sigma}\cdot\vec{a} + \lambda_2\,\varepsilon\,\vec{\sigma}\,\vec{a}\,\varepsilon = \lambda_1\vec{\sigma}\cdot\vec{a} + \lambda_2(\varepsilon^+-\varepsilon^-)\,\vec{\sigma}\cdot\vec{a}\,(\varepsilon^+-\varepsilon^-)$$

$$= \lambda_1\vec{\sigma}\cdot\vec{a} + \lambda_2\,\varepsilon^+\vec{\sigma}\cdot\vec{a}\,\varepsilon^+ + \lambda_2\,\varepsilon^-\cancel{\vec{\sigma}\,\vec{a}\,\varepsilon^-} - \lambda_2\,\varepsilon^+\vec{\sigma}\,\vec{a}\,\varepsilon^-$$

$- \lambda_2\,\varepsilon^-\vec{\sigma}\,\vec{a}\,\varepsilon^+$

$\frac{1-\varepsilon}{2}\,\sigma\cdot a\,\frac{1+\varepsilon}{2}$

$\begin{cases} T = 0 & - \quad 1 \text{ real.} \\ T = 1 & , \quad \text{one real one complex.} \end{cases}$

$T = \frac{1}{2}$, 2 complex. $= (0+1)$.

$\begin{cases} T = 1 & : \quad 1 \text{ real}, 1 \text{ complex.} \\ T = 2 & : \quad 1 \text{ real}, 2 \text{ complex.} \end{cases}$

$T = \frac{3}{2}$. 4 complex $= (1+2)$.

4 complex states p, p^c, n, n^c

or $p_L, \hat{p}_R, n_L, \hat{n}_R$ can be regarded as components of $T = \frac{3}{2}$.

charge x group $q = I_3$ | Q and N are conserved | Q & $N = \frac{\omega}{2}$ $Q = \omega$. $N = Q$

$e^{i\frac{\omega}{2}}$ $p_L = e^{iQ} e^{i\frac{Q}{2}}$ | $e^{iQ} e^{iN}$ | $e^{i3\frac{\omega}{2}}$

$n_L = -e^{-i\frac{Q}{2}}$ | $-e^{iN}$ | $e^{i\frac{\omega}{2}}$

$\hat{n}_R = e^{i\frac{Q}{2}}$ | e^{-iN} | $-e^{-i\omega}$

$\hat{p}_R = e^{-iQ} e^{-i\frac{Q}{2}}$ | $e^{-iQ} e^{-iN}$ | $e^{-3i\frac{\omega}{2}}$

$$\begin{pmatrix} -n_L & p_L \\ \hat{n}_R & n_L \\ \hat{n}_R & n_L \\ \hat{p}_R & -\hat{n}_R \end{pmatrix} \qquad \begin{pmatrix} -n & p \\ n^c & n \\ n^c & n \\ p^c & -n^c \end{pmatrix} = \text{spin } \frac{3}{2} \to e^{i(\vec{\rho}+\vec{\sigma})\cdot\frac{\vec{\omega}}{2}} \begin{pmatrix} -n & p \\ n^c & n \\ n^c & n \\ p^c & -n^c \end{pmatrix} e^{-i\vec{\sigma}\cdot\frac{\vec{\omega}}{2}}.$$

n, n^c, p^c

$n, (n^c + n^c), p^c$ form a triplet

$n_L, (\hat{n}_R + \hat{n}_R), \hat{p}_R$ form a triplet

n, n, p^c.

also n, n^c, p^c form a triplet

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10 bis ~~14 bis~~

if we add $i\,\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{b}\,\frac{3-\varepsilon}{2}$ to spin 2 we obtain the general form for spin $\frac{3}{2}$. $T^{\frac{3}{2}}: \frac{1+\varepsilon}{2}\vec{\sigma}\cdot\vec{a}\,\frac{3-\varepsilon}{2}\,N_3$

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{b}\,\frac{3-\varepsilon}{2} = \frac{1}{4}\Big[(\sigma b+\varepsilon\sigma b)(3-\varepsilon)\Big] = \frac{1}{4}\Big[3\sigma b+3\varepsilon\sigma b-\sigma b\varepsilon-\varepsilon\sigma b\varepsilon\Big]$$

$$= \frac{1}{4}\Big[(3\sigma-\rho)\cdot b+(3\rho-\sigma)\varepsilon\cdot b\Big]$$

$$\sigma\cdot(\rho\cdot\sigma) = \rho - i\rho\times\sigma$$

$$\sigma\varepsilon = \sigma\frac{1+\rho\cdot\sigma}{2} = \frac{1}{2}(\sigma+\sigma\cdot\rho\cdot\sigma) = \frac{1}{2}\Big[\rho+\sigma-i\rho\times\sigma\Big]$$

$$\rho\varepsilon = \rho\frac{1+\rho\cdot\sigma}{2} = \frac{1}{2}(\rho+\rho\cdot(\rho\cdot\sigma)) = \frac{1}{2}\Big[\rho+\sigma+i\rho\times\sigma\Big]$$

$$(3\rho-\sigma)\varepsilon = \frac{3}{2}\Big[\rho+\sigma+i\rho\times\sigma\Big]-\frac{1}{2}\Big[\rho+\sigma-i\rho\times\sigma\Big]$$

$$= \rho+\sigma+2i\rho\times\sigma$$

$$(3\sigma-\rho)+(3\rho-\sigma)\varepsilon = 3\sigma-\rho+\rho+\sigma+2i\rho\times\sigma = 4\sigma+2i\rho\times\sigma$$

$$\frac{1+\varepsilon}{2}\,\vec{\sigma}\cdot\vec{b}\,\frac{3-\varepsilon}{2} = \left(\vec{\sigma}+\frac{1}{2}i\,\vec{\rho}\times\vec{\sigma}\right)\cdot\vec{b}$$

2×4 matrix

$\rho\cdot\sigma\ \frac{1+\rho_3}{2}$

$(\vec{\sigma}-\frac{1}{2}\vec{\rho}\times\vec{\sigma})\,\frac{1-\rho_3}{2}$

$\rho^+\ \frac{1-\rho_3}{2}$

$\rho^+\sigma^+$, etc.

contains with spin

1 spin 0

3 spin 1, spin 2

2 spin $\frac{1}{2}$

4

$$\frac{1}{2}i\vec{\rho}\times\vec{\sigma}\cdot b = \begin{pmatrix} & -\frac{b^+}{\sqrt{2}} & 0 \\ & b_3 & \frac{b^+}{\sqrt{2}} \\ & 0 & -\frac{b^+}{\sqrt{2}} \\ & \frac{b^-}{\sqrt{2}} & 0 \end{pmatrix} \qquad \vec{\sigma}\cdot\vec{b} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ b_3 & \sqrt{2}b^+ \\ \sqrt{2}b^- & -b_3 \end{pmatrix}$$

1 spin 0 1

1 spin 1 3

2 spin $\frac{1}{2}$ 4

2 spin $\frac{3}{2}$ 8 $\frac{1+\rho_3}{2}$ and $\frac{1-\rho_3}{2}$

16

$$(\vec{\sigma}\cdot\vec{b}+\frac{1}{2}\vec{\rho}\times\vec{\sigma}\cdot\vec{b}) = \begin{pmatrix} & -\frac{b^+}{\sqrt{2}} & 0 \\ & b_3 & \frac{b^+}{\sqrt{2}} \\ & b_3 & \frac{b^+}{\sqrt{2}} \\ & \frac{3b^-}{\sqrt{2}} & -b_3 \end{pmatrix}$$

is part of $T=\frac{3}{2}$ not part of $T=2$

$T=2$ is also part of $T=\frac{3}{2}$

$$\text{spin } \frac{3}{2} \quad \begin{pmatrix} -a^+ & a^{++} \\ a^0 & a^+ \\ a^0 & a^+ \\ a^- & -a^0 \end{pmatrix}$$

spin $\frac{3}{2}$: $\Big[\frac{1}{2}(\vec{\rho}\,\vec{\sigma}+\vec{\sigma}\,\vec{\rho})-\frac{\delta}{3}\vec{\rho}\cdot\vec{\sigma}\Big]\cdot\vec{a}+i(\vec{\sigma}+\frac{i}{2}\vec{\rho}\times\vec{\sigma})\cdot\vec{b}$

$(1,\rho)$ combined with a spin

$\rho\cdot\sigma$	$(1,\rho)$	$(\vec{\sigma}-\frac{i}{2}\vec{\rho}\times\vec{\sigma})$	$(\vec{\sigma}+\frac{1}{2}\vec{\rho}\times\vec{\sigma})$	$\frac{1}{2}(\rho\sigma+\sigma\rho)+\rho\sigma$
1 spin 0	2 spin $\frac{1}{2}$	1 spin 1	1 spin 1	1 spin 2
1	4	3	3	5

Combines as spin $\frac{3}{2}$

$$\gamma_\mu \partial_\mu \psi = m \psi + e^{\dots} \psi^G$$

$$\gamma_\mu \partial_\mu e^{\dots} \psi = m e^{\dots} \psi + e^{\dots} e^{\dots} (e^{\dots} \psi^G)$$

$$e^{\dots} \boxed{\gamma_\mu \partial_\mu \psi = e^{\dots} \psi^G}$$

$$\boxed{(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\psi^R = m \psi^{G}_{Q} \dots R}$$

$$\psi \to e^{i\gamma_5 \vec{\tau}\cdot\vec{\alpha}} \psi.$$

$$\psi^G \to e^{-i\gamma_5 \vec{\tau}\cdot\vec{\alpha}} \psi^G.$$

$$(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\psi R = m \psi^{G}_{\bar{R}} (R \phi R)$$

$$(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\psi^G = m \psi \ \bar{\phi}$$

$$(\gamma_\mu \partial_\mu + m)\begin{pmatrix}\chi \\ \zeta\end{pmatrix}$$

χ, ζ. Majorana spinors

$(i\gamma_5 \vec{\tau})$ ..

2 quaternions

θ_1

θ_2

$$\gamma_\mu \partial_\mu \psi = m e^{\gamma_5 \theta \dots}$$

$$\psi = \begin{pmatrix} n_1 & n_1 \\ n_2 & n_2 \end{pmatrix}.$$

$$(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\psi = m \sigma_2 \psi^*$$

$$= m \hat{\psi} e_2$$

$$\left[(-1)^{\theta} \partial_0 + \vec{e}\cdot\vec{\nabla}\right]\psi = 0$$

$$(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\psi = m \sigma_2 \psi^* \ \phi$$

$$(\partial_0 - \vec{\sigma}\cdot\vec{\nabla})\psi = m \psi^G \phi$$

$$(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\psi = m \hat{\psi} e_2.$$

$$= \boxed{(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\psi = m \sigma_2 \psi^*}$$

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$\rho_1 a_1 + \rho_2 a_2$

$a_1 + i a_2$

$a_1 + i a_2$

$\rho_2 = \begin{pmatrix} & i \\ -i & \end{pmatrix}$

$\frac{a_1 + i a_2}{\sqrt{2}} = a^+ \quad (\rho_2 + \sigma_2)\, a_2$

$$\left(\frac{\vec{\rho} + \vec{\sigma}}{2}\right) \cdot \vec{a} = \frac{1}{2}\begin{pmatrix} a_3 + a_3 & a_1 + i a_2 & a_1 + i a_2 & \\ a_1 - i a_2 & a_3 - a_3 & & a_1 + i a_2 \\ a_1 - i a_2 & & -a_3 + a_3 & a_1 + i a_2 \\ & a_1 - i a_2 & a_1 - i a_2 & -a_3 - a_3 \end{pmatrix} = \begin{pmatrix} a_3 & \frac{a^+}{\sqrt{2}} & \frac{a^+}{\sqrt{2}} & 0 \\ \frac{a^-}{\sqrt{2}} & 0 & 0 & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & 0 & 0 & \frac{a^+}{\sqrt{2}} \\ 0 & \frac{a^-}{\sqrt{2}} & \frac{a^-}{\sqrt{2}} & -a_3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & ib \\ ib & 0 \\ ib & 0 \\ c & -ib \end{pmatrix}$$

$$\frac{\vec{\rho} - \vec{\sigma}}{2} \cdot \vec{a} = \frac{1}{2}\begin{pmatrix} a_3 - a_3 & -a_1 + i a_2 & & \\ -(a_1 - i a_2) & a_3 + a_3 & & \\ & & -a_3 - a_3 & -(a_1 + i a_2) \\ & & -(a_1 - i a_2) & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \frac{a^+}{\sqrt{2}} & \frac{a^+}{\sqrt{2}} & 0 \\ -\frac{a^-}{\sqrt{2}} & a_3 & 0 & \frac{a^+}{\sqrt{2}} \\ \frac{a^-}{\sqrt{2}} & 0 & -a_3 & -\frac{a^+}{\sqrt{2}} \\ 0 & \frac{a^-}{\sqrt{2}} & -\frac{a^-}{\sqrt{2}} & 0 \end{pmatrix}$$

$\rho_2 \sigma_1$

$$\vec{\rho} \times \vec{\sigma} \cdot \vec{a} = \begin{pmatrix} 0 & 0 & i\frac{a_1 - a_2}{2} & 0 \\ & & -i a_3 & \frac{-i a_1 + a_2}{2} \\ i a_3 & 0 & i\frac{a_1 - a_2}{2} & \\ & & \frac{-i a_1 - a_2}{2} & 0 \end{pmatrix}$$

$$i \vec{\rho} \times \vec{\sigma} \cdot \vec{a} = \begin{pmatrix} & & -\frac{a_1 - i a_2}{2} & 0 \\ & & a_3 & \frac{a_1 + i a_2}{2} \\ & & 0 & -\frac{a_1 - i a_2}{2} \\ & & \frac{a_1 - i a_2}{2} & 0 \end{pmatrix}$$

$+i - i$

$\rho_1 \sigma_2 - \rho_2 \sigma_1$

$\rho_2 \sigma_3 - \rho_3 \sigma_2$

$\rho_3 \sigma_1 - \rho_1 \sigma_3$

$-i - i$

$\rho_2 = \begin{pmatrix} & i \\ -i & \end{pmatrix}$

$$\frac{1}{2}\begin{pmatrix} 0 & \sigma_2 - i\sigma_1 \\ \sigma_2 + i\sigma_1 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & & 0 & 0 \\ & & -i & 0 \\ & i & & \\ 0 & & & \end{pmatrix}$$

$$i \vec{\rho} \times \vec{\sigma} \cdot \vec{a} \quad \begin{pmatrix} & & -\frac{a^+}{\sqrt{2}} & 0 \\ & & a_3 & \frac{a^+}{\sqrt{2}} \\ & & 0 & -\frac{a^+}{\sqrt{2}} \\ & & \frac{a^-}{\sqrt{2}} & 0 \end{pmatrix}$$

$\frac{1+\varepsilon}{2}\, \rho \times \sigma \cdot \vec{a} =$

$$\frac{1}{2}\begin{pmatrix} -\sigma_2 & i\sigma_3 \\ -i\sigma_3 & \sigma_2 \end{pmatrix} = \begin{pmatrix} & -\frac{i}{2} & \frac{i}{2} & \\ \frac{i}{2} & & & -\frac{i}{2} \\ & & & \frac{i}{2} \\ -\frac{i}{2} & +\frac{i}{2} & & -\frac{i}{2} \end{pmatrix}$$

$$-i \vec{\rho} \times \vec{\sigma} \cdot \vec{a} \quad \begin{pmatrix} & \frac{a^+}{\sqrt{2}} & 0 \\ & -a_3 & \frac{a^+}{\sqrt{2}} \\ & 0 & \frac{a^+}{\sqrt{2}} \\ & \frac{a^-}{\sqrt{2}} & 0 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} \sigma_1 & -\sigma_3 \\ -\sigma_3 & -\sigma_1 \end{pmatrix} = \begin{pmatrix} & \frac{1}{2} & -\frac{1}{2} & \\ \frac{1}{2} & & & +\frac{1}{2} \\ -\frac{1}{2} & & & -\frac{1}{2} \\ & +\frac{1}{2} & -\frac{1}{2} & \end{pmatrix}$$

$$\vec{\rho} - \vec{\sigma} - i\vec{\rho} \times \vec{\sigma} \cdot \vec{a} = \begin{pmatrix} & \sqrt{2}a^+ & 0 \\ & a_3 & 0 \\ & -a_3 & 0 \\ & -\sqrt{2}a^- & 0 \end{pmatrix}$$

$$(\vec{\rho} - \vec{\sigma} + i\vec{\rho} \times \vec{\sigma})\vec{a} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & \sqrt{2}a^+ \\ & & -a_3 & -\sqrt{2}a^- \\ & & 0 & 0 \end{pmatrix}$$